

The Endogenous Longevity Channel: Health Investments, Aggregate Savings, and Macroeconomic Policy*

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Abstract

This paper studies how an endogenous longevity channel, the partial control of life expectancy through preventive health investment, shapes macroeconomic outcomes and policy design. I first study how health investments affect individual saving decisions in a two-period model where survival probabilities depend on prevention. Endogenous longevity creates a non-linear asset-income relationship: health spending depresses saving and yields concavity at low incomes, while longer expected lifespans raise the value of saving and generate convexity at higher incomes. Embedding this mechanism into a neoclassical OLG model, I then examine its implications for macroeconomic policy design. When annuity markets are incomplete, individuals who extend their lifespans fail to internalize the social costs associated with longer survival. Thus, under an optimal policy regime, efficiency requires a positive tax on health investment. In more realistic second-best settings, however, health subsidies become optimal, as they both improve efficiency by compensating for annuity imperfections and enhance equity by narrowing longevity and income inequality. I then build a quantitative model disciplined by U.S. data to assess two applications: (i) how the endogenous longevity channel alters the contribution of demographic and inequality trends to the rise in aggregate saving and decline in real interest rates over the past five decades, and (ii) the welfare effects of alternative policy reforms in preventive health subsidies.

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1 Introduction

Adult life expectancy has increased markedly in advanced economies over the past half-century, but these gains have not been evenly shared. Access to, and returns from, medical innovation differ substantially across socioeconomic groups, generating persistent disparities in longevity.¹ These disparities suggest that longevity is at least partly endogenous, shaped by health-related decisions and economic circumstances that influence how households allocate resources over the life cycle. Understanding the implications of this endogeneity requires a framework in which longevity itself is an outcome of health-related choices. In this paper, I study such a framework, focusing on how health and financial investments jointly shape individual behavior and aggregate outcomes, as well as their implications for the design of policy.

To ground this idea empirically, I draw on the large body of evidence documenting socioeconomic disparities in healthcare usage and construct a proxy for preventive health investment. By preventive investment I mean non-acute medical use aimed at maintaining or improving health, rather than treating existing conditions. Using the Medical Expenditure Panel Survey (MEPS), a nationally representative dataset on U.S. medical spending, I measure preventive investment as medical spending among respondents who consistently report good health and no recent deterioration, under the premise that their expenditures better reflect routine, health-maintaining use. Using this measure, I find that the top 10 percent spends about twice as much as the bottom 90 percent up to roughly age 75; thereafter, the difference narrows but remains positive.

I then formalize the endogenous longevity channel, i.e., the idea that longevity is partly determined by individuals' health investments and the resources they can allocate to them. In this framework, prevention constitutes the primary form of health investment because it directly raises the likelihood of surviving into later periods. Building on this mechanism, I study both the positive and normative consequences of endogenous longevity. On the positive side, I examine how health investments influence individual and aggregate saving, and on the normative side, I analyze how the endogeneity of longevity shapes the design of government policy.

How do health investments influence individual saving decisions? To address this question, I study an individual consumption–saving problem extended to include endogenous longevity, meaning that the individual lives up to two periods with a survival probability that depends on their health investments. In the first period, they work and allocate income between consumption, health investment, and financial saving; if they survive, they consume in the second period. The baseline analysis abstracts from annuities, though I discuss how their inclusion would affect the main results.²

¹Chetty et al. (2016) document that at age 40, individuals in the top 1 percent of the income distribution live roughly 15 years longer than those in the bottom 1 percent, and that this longevity gap has widened over the past two decades. Other studies documenting persistent and widening disparities in life expectancy across income and socioeconomic groups include Pappas et al. (1993), Adler and Ostrove (1999), Deaton and Paxson (2001), Case and Deaton (2015), Cutler and Lleras-Muney (2010), and Bound et al. (2015).

²Private annuitization is limited in practice: few retirees purchase life annuities and most defined-contribution plans neither offer nor default workers into them; see Brown (2007) and Benartzi et al. (2011). Empirical work points to adverse selection and other frictions—e.g., Finkelstein and Poterba (2004) document selection in the U.K. annuity market—despite money's-worth measures that are often close to actuarially fair before loads; see Mitchell et al. (1999).

Moreover, there is no government intervention. In this tractable model, I characterize (i) the curvature of the asset policy as a function of (permanent) income, and (ii) the sensitivity of asset decisions with respect to interest rates and (exogenous) survival rate movements.

Under standard macroeconomic assumptions, exogenous life expectancy and homothetic preferences over consumption, savings decisions are linear in income in my framework.³ Departing from this benchmark, I show that endogenous longevity affects this relationship through two opposing effects. On one hand, health investment behaves as a luxury good: as income rises, individuals devote more to health, causing asset accumulation to increase at a decreasing rate. This mechanism produces a concave asset–income relationship. On the other hand, when the longevity gains from health investment are sufficiently strong, the longer expected lifetime raises the marginal utility of future consumption, inducing higher saving at higher income levels. In this case, the asset policy becomes convex. The overall curvature therefore depends on the relative strength of these forces, specifically, on how the elasticity of survival with respect to health spending interacts with the degree of utility satiation in consumption. Under general conditions, I show that the asset policy is concave at low income levels and convex beyond a threshold that depends on the “productivity” of health investment and the speed of satiation in consumption.

Endogenous longevity also affects the elasticity of individual saving with respect to exogenous changes in survival or interest rates. In a standard setting with exogenous longevity, higher life expectancy or a higher real interest rate mechanically increases asset demand, as agents plan for longer lifespans or intertemporal substitution of consumption.⁴ When longevity is instead an outcome of health investment, the response of saving depends on how such changes alter the incentives to invest in health. If an exogenous improvement in survival technology reduces the marginal return to health spending (e.g., because these advances allow individuals to live longer with the same spending) agents reallocate resources from health to financial assets. In this case, the responsiveness of saving to longer lifespans is amplified, as individuals reinforce the direct effect of higher survival by saving more. Conversely, if the same improvement increases the productivity of health investment (i.e., making additional spending on health more effective) agents substitute toward health, and the saving response is dampened.

A similar reasoning applies to changes in the interest rate: when longevity gains from health investment are large, higher returns to saving mainly raise asset accumulation, increasing the elasticity of saving to interest rate movements; when these gains are limited, part of the additional resources is diverted toward health. Overall, compared to a framework with exogenous longevity, the model implies changes in the sensitivity of individual savings to demographic or interest rate: Saving responds more strongly when the reallocation toward financial assets dominates (crowding-in case) and more weakly when health investment absorbs a larger share of income (crowding-out case).

³This linearity in permanent income holds in models adhering to the permanent income hypothesis as well as in canonical precautionary-saving models (see [Aiyagari \(1994\)](#)), which are well known to generate concave consumption functions (i.e., convex asset functions) in current income or liquid assets (see [Carroll and Kimball \(1996\)](#)), but predict linear asset function in income. [De Nardi \(2004\)](#) and [Straub \(2019\)](#) are exceptions that introduce non-homothetic preferences over consumption in an otherwise standard consumption-saving problem.

⁴Because old-age income is absent in the baseline model, the interest rate effects are completely driven by the intertemporal substitution effect.

These features of individual asset behavior have direct implications for the general equilibrium effects of inequality changes and demographic. To approach these general equilibrium effects, I embed the studied individual consumption-saving problem into a two-period neoclassical overlapping-generations (OLG) model where households differ in their labor productivity, which is exogenously allocated for new borns. I study the laissez-faire equilibrium in this OLG model. A mean-preserving spread in the income distribution increases (decreases) aggregate saving and lowers (raises) the equilibrium real interest rate when the longevity gains from health investment are sufficiently large (small). Intuitively, when income heterogeneity translates strongly into longevity heterogeneity, inequality amplifies the aggregate saving motive; when longevity is relatively inelastic to income, inequality dampens it.

The macroeconomic effects of longer life expectancy depend on whether the improvement reflects an exogenous rise in survival probabilities or a change in the productivity of health investment. By altering the elasticity of saving with respect to longevity and the interest rate, endogenous longevity modifies the slope of the aggregate saving schedule and, consequently, the magnitude of the interest rate adjustment required to clear asset markets. Because fertility is exogenous in my model and does not affect individual behavior directly, its equilibrium effect on real interest rates operates entirely through the change in the interest rate elasticity of savings. The relative contributions of demographic and inequality shocks to aggregate saving and interest rate dynamics are examined quantitatively in a calibrated quantitative model.

I next turn to the policy implications of the endogenous longevity channel. This analysis is relevant for at least two reasons. First, the channel provides a mechanism through which the design of health and pension systems can influence macroeconomic outcomes. Second, the historical and prospective effects of demographic and inequality trends on aggregate saving and real interest rates ultimately depend on the policy environment in which they unfold. A full assessment of the role of endogenous longevity therefore requires analyzing how optimal policy design changes when health investment becomes an endogenous household decision margin. To this end, I study both first-best and second-best allocations under alternative policy regimes, comparing efficient benchmarks with settings that incorporate fiscal and market imperfections. The analysis pays particular attention to a central policy question: whether governments should subsidize or tax health investment in order to balance efficiency and redistribution objectives.

In the first-best allocation, within-generation consumption choices are determined by the marginal return on capital and are independent of individual survival rates. The social planner internalizes not only the individual effects of survival, through its contribution to expected lifetime utility, but also its aggregate implications for the economy, as longevity affects the dependency ratio and the demographic composition of the labor force. Comparing the first-best allocation to the laissez-faire outcome reveals two distinct wedges: one in intertemporal consumption–saving decisions and another in health-investment decisions. The first wedge arises because, in the absence of actuarially fair annuities, households cannot fully insure idiosyncratic mortality risk. Consequently, the effective return on financial assets falls short of the annuity-adjusted efficient return. The second wedge concerns health investment: without fair and complete annuities, individuals are not penalized for

living longer as a result of higher health spending. In equilibrium, they invest excessively in health relative to the social optimum, failing to internalize the fiscal and demographic consequences of increased longevity.

Implementing the first-best allocation calls for a set of complex policy instruments and market mechanisms, including redistributive transfers, actuarially fair annuity contracts, and a penalization of individual health investment. If annuity markets are competitive and can condition contracts on health behavior, this penalization could, in principle, be incorporated directly into annuity pricing. Such contracts, however, would demand detailed monitoring of individual health investments, a technology that is unlikely to exist in practice. Given this limitation, the implementable counterpart is a Pigouvian tax on health investment. Although taxing health may appear counterintuitive, its rationale is straightforward: when longer lifespans result from higher private health spending without corresponding adjustments in annuity pricing, individuals fail to internalize the fiscal and demographic costs of increased longevity, effectively shifting part of the burden onto future cohorts. A Pigouvian tax corrects this externality by aligning private and social incentives. The direction of the optimal tax, whether it rises or falls with advances in survival technology, depends on how those advances affect equilibrium health-investment behavior.

A further implication of the first-best allocation is worth emphasizing. When intergenerational Pareto weights are held constant, the aggregate capital stock and the equilibrium real interest rate remain unchanged in response to exogenous improvements in survival technology. This neutrality result is not universal, it depends on the planner's distributional weights across cohorts, but it underscores an important point: in an efficient allocation, demographic change alone does not generate secular-stagnation dynamics. Such outcomes arise primarily from market imperfections that distort saving and health-investment behavior rather than from longevity gains themselves.

I next analyze optimal policy in a more constrained, second-best environment. Suppose that informational frictions make annuity markets imperfect, leading to pooling equilibria or their complete absence, and that the government's instruments are limited to capital and health taxation. In this setting, the optimal policy involves subsidizing health investment. From an efficiency perspective, the subsidy alleviates distortions created by incomplete annuity markets: by raising life expectancy, greater health investment partially substitutes for the missing insurance. From a redistributive standpoint, the subsidy compresses longevity differentials across income groups and reduces inequality in consumption during old age.

Overall, the policy implications of demographic and inequality trends depend on the interaction between two forces: the magnitude and direction of the demographic "externality," and the strength of redistribution and completing-the-market motives. Although theory does not yield a universal ranking of these forces, the quantitative analysis indicates that, under a wide range of plausible demographic and inequality scenarios, health-investment subsidies enhance welfare. This rationale differs from standard arguments based solely on equity or the underuse of preventive care: in this framework, the case for subsidizing health investment arises from incomplete annuity markets and

the general-equilibrium effects of endogenous longevity.⁵

I next turn to a quantitative overlapping-generations (OLG) life-cycle model to assess the quantitative relevance of the endogenous longevity channel in a richer environment. The model features heterogeneous agents who live for multiple periods, face borrowing constraints, and make endogenous health-investment decisions that influence their survival probability, productivity, curative medical expenditures, and utility. The government intervenes through a pay-as-you-go pension system, taxes on capital and health investment, coinsurance for curative medical expenditures, and a progressive labor-income tax schedule.

The model is calibrated to U.S. data, primarily using the Medical Expenditure Panel Survey (MEPS) and the Health and Retirement Study (HRS). Calibration targets include the life-cycle profile of preventive health spending, the distribution of income and wealth, and key features of the tax system. With this structure in place, I conduct two main quantitative exercises. First, I evaluate how the endogenous longevity channel modifies the contribution of demographic change and rising inequality to the long-run decline in real interest rates. Second, I examine the welfare and distributional consequences of alternative policies, focusing on health-investment subsidies and the size of social-security benefits.

The quantitative results indicate that accounting for endogenous longevity substantially reduces the contribution of inequality to the observed decline in equilibrium interest rates. In a model without endogenous longevity, the increase in income inequality accounts for roughly a 100-basis-point fall in the real rate; once health investment endogeneity is introduced, this contribution falls to about 40–50 basis points, depending on the specification. The smaller effect reflects a flatter asset-policy function with respect to income—endogenous longevity makes savings less convex because higher-income individuals allocate part of their additional resources to health rather than financial assets. The reduction is larger when the model includes both curative medical expenses and the productivity effects of health investment.

The model also shows that exogenous improvements in survival amplify the macroeconomic role of the endogenous longevity channel. When longevity gains arise from technological progress that increases survival directly, individuals can maintain longer, healthier lives without proportionally higher health spending. This strengthens the aggregate saving response to demographic change. The magnitude of this amplification, however, depends on how medical costs evolve with age. When curative medical expenses rise steeply at older ages, additional longevity encourages higher health spending, which partially offsets the saving motive by shifting resources toward maintaining

⁵Beyond the mechanism emphasized in this paper, missing annuity markets and the general-equilibrium effects of endogenous longevity, the literature provides several other rationales for encouraging prevention. Among some of these other rationales are: (i) healthier individuals work longer and more efficiently, raising aggregate labor supply and fiscal capacity (Bloom et al., 2014; Cutler and Lleras-Muney, 2010; French and Jones, 2011); (ii) private insurers underprovide prevention with long-run payoffs because of turnover and limited contract duration (Arrow, 1963; Pauly, 1968; Einav and Finkelstein, 2018); (iii) prevention reduces public expenditures on disability and late-life care and increases tax revenues (Cutler and Zeckhauser, 2000; Gruber and Wise, 2004); (iv) vaccination and screening generate herd-immunity benefits not internalized by individuals (Geoffard and Philipson, 1997; Francis, 1997); and (v) Behavioral and informational frictions: bounded rationality, present bias, and low salience lead to underinvestment in prevention even when privately profitable (O'Donoghue and Rabin, 1999; Thaler and Sunstein, 2008; Baicker et al., 2010).

productivity and reducing late-life medical costs.

Finally, the welfare analysis suggests that subsidizing health investment raises welfare across successive generations. A positive health-investment subsidy reduces both longevity and wealth inequality while also increasing aggregate saving. Expanding social-security benefits financed by lower health-investment taxation yields further welfare gains, highlighting the joint role of pension and health policies in shaping the efficiency and equity of long-run outcomes.

Literature Review. This paper relates to four strands of literature. The first branch studies individual behavior and health-related decisions over the life cycle. Early contributions such as [Banks et al. \(2001\)](#), [Bloom et al. \(2007\)](#), [De Nardi et al. \(2009\)](#), and [Nardi et al. \(2010\)](#) analyze how health status and medical spending influence household saving, retirement behavior, and wealth accumulation, more recent work—including [Foltyn and Olsson \(2021\)](#), [Blundell et al. \(2024\)](#), and [Hosseini et al. \(2025\)](#)—incorporates heterogeneity in health shocks, preferences, and expectations to explain the joint evolution of health, consumption, and wealth in old age. This paper contributes by providing new empirical evidence on life-cycle preventive health investment gradients by income, linking those gradients to wealth and survival profiles using MEPS, HRS, and ATUS, and showing that income-driven differences in prevention generate systematic differences in saving horizons, thereby shaping the slope and curvature of asset policies over the life cycle.

The second branch develops models of endogenous health formation, in which individuals accumulate health capital through preventive investments. The seminal contribution of [Grossman \(1972\)](#) formalized health as both a consumption and an investment good, more recent studies—such as [Cole et al. \(2019\)](#), [Margaris and Wallenius \(2023\)](#), [Capatina and Keane \(2023\)](#), [Ozkan \(2024\)](#), and [Mahler and Yum \(2024\)](#)—extend this framework to explore the macroeconomic and distributional implications of health investment, health inequality, and longevity dynamics. This paper contributes by embedding endogenous longevity directly into an overlapping-generations environment, characterizing the conditions under which income–health complementarities make saving either convex or concave in income, and tracing the equilibrium consequences for aggregate saving and the real interest rate, in particular, I show that the response of aggregate saving to exogenous longevity improvements depends critically on whether those improvements raise or lower the marginal effectiveness of prevention, a mechanism that helps reconcile heterogeneous macro responses to demographic shifts.

A growing macro literature argues that longer lives reshape equilibrium outcomes—most visibly through their effects on individual saving behavior and aggregate saving.

The third branch examines disparities in health outcomes across socioeconomic groups. Foundational research by [Adler et al. \(1994\)](#), [Deaton and Paxson \(1998\)](#), [Lindahl \(2005\)](#), [Lleras-Muney \(2005\)](#), and [Cutler and Lleras-Muney \(2008\)](#) documents the strong and persistent relationship between socioeconomic status and health, subsequent studies—including [Adda et al. \(2009\)](#), [Deaton \(2013\)](#), [Chetty et al. \(2016\)](#), [Erixson \(2017\)](#), [Schwandt \(2018\)](#), [Davies et al. \(2018\)](#), [Case and Deaton \(2020\)](#), [De Nardi et al. \(2024\)](#) and [Borella et al. \(2024\)](#)—quantify the evolution and mechanisms of these gradients. This paper contributes by documenting a robust income gradient in preventive medical spending and time devoted to health-promoting activities, showing that these gradients persist across

ages and help explain observed gaps in survival and wealth, and by constructing a synthetic age–income panel that jointly links income, prevention, health status, longevity, and assets, providing targeted moments for calibrating the model’s longevity–income channel.

Finally, the fourth branch focuses on the design of social security and intergenerational transfer systems. Classical analyses by [Samuelson \(1975b\)](#), [Diamond \(1977\)](#), [Sheshinski and Weiss \(1981\)](#), [Smith \(1982\)](#), and [Boadway et al. \(1991\)](#), and more recently by [Krueger and Kubler \(2006\)](#) and [Hosseini \(2015\)](#), characterize efficient pension and annuity arrangements under different demographic and market structures. Quantitative and empirical extensions—such as [İmrohoroglu et al. \(1995\)](#), [Poterba et al. \(2007\)](#), [Conesa and Garriga \(2008\)](#), [Conesa and Garriga \(2009\)](#), and [Poterba \(2014\)](#)—study how alternative designs of social security and health systems affect savings, welfare, and fiscal sustainability. This paper contributes by deriving first-best and second-best policy prescriptions when longevity is endogenous and annuity markets are imperfect, showing that in first best health investment must be penalized through health-contingent annuities or Pigouvian taxes to internalize the demographic pecuniary externality, and demonstrating that in realistic second-best environments with limited instruments, subsidizing prevention can be optimal on efficiency and redistribution grounds, the calibrated model then quantifies how these policies mediate the effects of demographic and inequality trends on aggregate saving and the real interest rate.

Outline. The remainder of the paper is organized as follows. In [Section 2](#), I document stylized facts on the life-cycle patterns of preventive healthcare investment and their relationships with longevity and wealth. [Section 3](#) develops a theoretical framework that formalizes the positive implications of the endogenous longevity channel, highlighting how income-driven health investment shapes individual saving behavior and macroeconomic equilibria. [Section 4](#) analyzes the normative dimension of the model by comparing first-best and second-best allocations under alternative policy regimes, focusing on the efficiency and redistributive roles of health-related taxation and subsidies. [Section 5](#) introduces the quantitative overlapping-generations model and describes its calibration to U.S. data. [Section 6](#) presents the quantitative findings, evaluating how endogenous longevity amplifies the effects of demographic and inequality trends on aggregate saving, real interest rates, and welfare. Finally, [Section 7](#) concludes with a discussion of the broader implications for macroeconomic policy and future research.

2 Motivating Facts: Unequal Health Investments in the US

This section provides motivational facts in health, survival, medical spending, health insurance, and time use across socioeconomic groups in the United States. The analysis focuses on systematic differences in preventive health investments by income and age, highlighting gradients that guide the model’s calibration and inform its core mechanisms. These stylized facts motivate the modeling framework.

2.1 Data Sources

The microdata used in this paper come from two nationally representative U.S. surveys: the Medical Expenditure Panel Survey (MEPS) and the Health and Retirement Study (HRS). The MEPS provides detailed information on total healthcare expenditures at the individual level, including out-of-pocket spending and reimbursements from private insurance, Medicaid, Medicare, and other public or private sources. The HRS complements the MEPS by providing data on longevity, wealth, and multiple measures of health status.

I group individuals into five-year age bins, covering ages 25 to 90, and classify them within each bin according to their position in the household income distribution. I apply the same age–income classification to both datasets, summarizing key variables at each age–income cell. Further details on all data sources and variable definitions are provided in [Appendix A](#).

2.2 Preventive Healthcare

Figure 1 presents evidence from the MEPS on the relationship between health status and preventive healthcare expenditures across income groups over the life cycle, using data from MEPS. Preventive care is measured here through pecuniary (monetary) health investments, focusing on medical spending among individuals who are in good health or who successfully maintain or improve their health status across consecutive survey years. Each panel reports average expenditures for males in the top 10 percent and bottom 90 percent of the income distribution.

Panel (a) displays the fraction of individuals reporting good or very good health by five-year age groups. The share in good health declines steadily with age for both groups, yet remains consistently higher among higher-income individuals. The gap is already visible in the late twenties and widens with age, illustrating that health–income gradients are a persistent feature across the life course, not only among older adults as seen in HRS data. These disparities likely reflect cumulative differences in access to preventive care, work conditions, and lifestyle factors that compound over time.

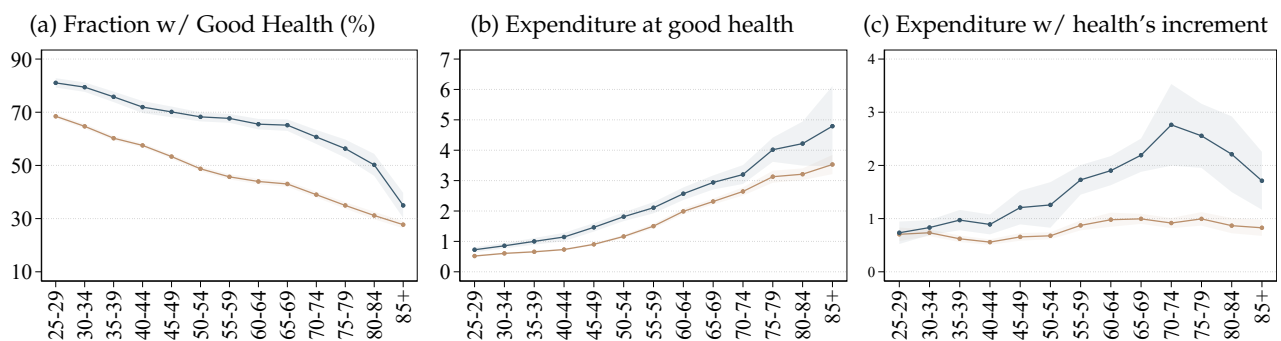
Panel (b) shows total medical expenditures among individuals reporting good health (“excellent” or “very good”) in a given year. Because these individuals are not in poor health, their spending is more likely to reflect preventive healthcare activities—such as routine checkups, diagnostic screenings, and other forward-looking medical investments—rather than treatment for acute conditions. The figure reveals that these expenditures increase steadily with age, particularly after midlife. While both income groups exhibit upward-sloping profiles, higher-income individuals consistently spend more at all ages. The gap widens in later adulthood, reaching roughly \$3,000–\$4,000 at ages 75 and above, suggesting that richer individuals maintain higher levels of preventive care even when in good health.

Panel (c) provides a complementary measure based on changes in self-reported health across consecutive waves. Because MEPS interviews each household five times over a two-year period, it allows tracking of respondents’ health status between periods t and $t + 1$. Individuals who report

being in good health at time t and who maintain or improve that status at $t + 1$ are classified as preventers, and their total medical spending defines preventive health expenditures. The resulting profiles show pronounced income gradients: individuals in the top income group spend more at nearly all ages, with the largest disparities occurring around retirement age (65–74), when high-income males spend roughly \$1,500 more per year on preventive care than their lower-income counterparts.

Overall, Figure 1 indicates that preventive medical spending rises with age and is systematically higher for richer individuals, even when conditional on good health or stable health trajectories. This suggests that income-related differences in proactive health investments emerge well before the onset of major health deterioration and likely contribute to the persistent health and longevity gaps documented in the earlier figures. These results provide micro-level evidence consistent with the model’s mechanism in which higher-income households invest more in health capital, thereby achieving longer expected lifespans and greater wealth accumulation over the life cycle.

Figure 1: Health Status and Preventive Healthcare



Note. Expenditure variables are in thousands USD. Income groups: Top 10% percent and Bottom 90% percent.

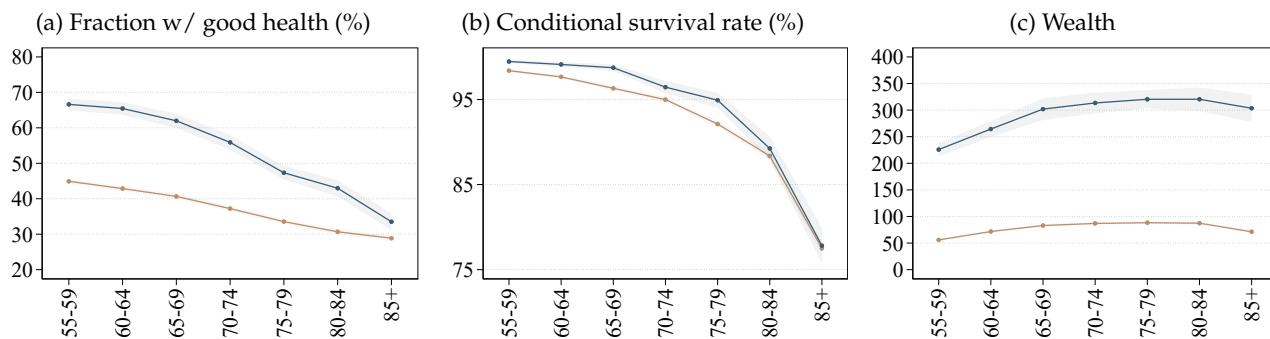
2.3 Longevity, Health and Wealth

Figure 2 displays descriptive evidence from the Health and Retirement Study (HRS) on the relationship between health status, survival, and wealth across income groups among U.S. males aged 55 and older. The figure plots average outcomes for two income categories—the top 10 percent and the bottom 90 percent—constructed from the RAND Harmonized HRS between 1992 and 2020. The top-income group represents high-earnings households, while the bottom group captures the remaining population, providing a simple yet informative contrast between individuals at different positions of the income distribution.

Panel (a) shows the fraction of individuals reporting good or very good health by five-year age groups. The share in good health declines monotonically with age for both groups but remains markedly higher for high-income males at every age. At ages 55–59, around two-thirds of high-income individuals report good health, compared with fewer than half among the bottom 90 percent. By ages 80 and older, these shares fall to roughly 40 percent and 30 percent, respectively. This persistent gap underscores the strong gradient between income and self-reported health that widens slightly with

age, consistent with cumulative differences in preventive care, health behaviors, and medical access.

Figure 2: Longevity, health and wealth



Note. Wealth is in thousands USD. Income groups: Top 10% percent and Bottom 90% percent.

Panel (b) depicts the conditional survival rate, defined as the probability of surviving to the next five-year age group conditional on reaching the current one. Survival probabilities exceed 95 percent at ages 55–69 for both groups but begin to diverge sharply thereafter. By age 80, the high-income group exhibits survival rates roughly 3–5 percentage points higher than the rest of the population, and the difference widens further in the oldest age groups. These patterns reflect the well-documented longevity gradient by socioeconomic status, consistent with findings in Chetty et al. (2016) and other U.S. mortality studies.

Panel (c) presents average household wealth (in thousands of USD), which rises during the pre-retirement years and peaks around ages 70–79 before gradually declining. High-income households accumulate substantially greater wealth at every age—roughly four to five times that of the bottom 90 percent—and experience slower decumulation late in life. Although differences in wealth are not entirely explained by longevity disparities, they are strongly correlated with the patterns observed in health and survival: longer life expectancy among wealthier individuals increases incentives for saving and amplifies wealth accumulation over the life cycle.

Taken together, data reveals a consistent association between economic status, health, and longevity. Higher-income males not only exhibit better health and longer survival but also sustain significantly greater wealth throughout retirement. These patterns motivate the paper’s modeling framework, in which endogenous health investments create a channel linking income inequality to differences in life expectancy and, ultimately, to aggregate outcomes such as saving behavior and equilibrium real interest rates.

3 A Simple Model of Endogenous Longevity

In this section, I characterize the role of endogenous longevity in a tractable model where individuals live up to two periods and influence their likelihood to survive to the second period through health investments. For simplicity, I abstract from government intervention and study the resulting

laissez-faire equilibrium. I begin by examining how the individual saving decision is affected by the endogenous longevity channel. Building on this analysis, I then explore how endogenous longevity shapes the partial and general equilibrium effects of inequality and demographic change in the laissez-faire economy.

3.1 Two-Period Consumption-Investment Problem

Consider an individual who lives for a maximum of two periods in an economy with only one final good. The individual's preferences are separable across time and given by

$$u(c_y) + \Psi(h) \cdot u(c_o)$$

where c_y and c_o denote consumption when young and old, respectively. Consumption in each period generates utility according to the same utility function, $u(\cdot)$. The individual survives to the second period with probability $\Psi(h)$, where h denotes her health investment chosen in the first period. Throughout the document, I assume that u and Ψ satisfy regular, standard conditions.⁶ To highlight the role of endogenous longevity, I abstract from other channel through which health investments can affect individual decisions (e.g., health-dependent utility).⁷

In the first period, the individual receives income y and allocates it between financial saving a and health investment h , consuming the remainder $c_y = y - a - h$. In the second period, if she survives, she consumes the gross return on her savings. As I assume that the individual has access only to a riskless bond with gross return R and that there is no social security program, old-age consumption is $c_o = R \cdot a$. Thus, the optimization problem is

$$\max_{a, h} u(y - a - h) + \Psi(h) \cdot u(Ra). \quad (1)$$

The key object of analysis in this section is the asset policy $a(y)$ and, to characterize it, I rewrite the problem as a two-stage problem. The first problem is an intertemporal one determining an optimal wealth level as a function of income $\omega(y)$. This wealth, however, can take the form of financial savings or health investments. The second problem is a static portfolio decision determining the optimal financial investments as a function of total wealth $\alpha(\omega)$. Combining both stages yields the overall asset policy, $a(y) \equiv \alpha(\omega(y))$.

Intertemporal Problem. The first stage is an intertemporal problem that determines the optimal level of resources ω carried into the second period:

$$\max_{\omega} u(y - \omega) + V(\omega) \quad (2)$$

⁶These assumptions, which guarantee interior solutions, are (i) both functions are increasing concave functions: $u', \Psi' > 0$, $u'', \Psi'' < 0$; and (ii) Inada conditions: $\lim_{c \rightarrow \infty} u'(c) = \lim_{h \rightarrow \infty} \Psi'(h) = 0$, and $\lim_{c \rightarrow 0} u'(c) = \lim_{h \rightarrow 0} \Psi'(h) = \infty$.

⁷Households do not derive utility from bequests so that all bequests are accidental. This is a simplification and I discuss its consequences conceptually below and numerically in Section 6.

where $V(\omega)$ is the continuation value solving the second-stage portfolio problem (see (4)). The object of interest in this first stage is the total saving policy, $\omega(y)$, which satisfies the first-order condition,

$$u'(y - \omega) = V'(\omega). \quad (3)$$

What are the key properties of the total wealth function? Is it concave or convex as a function of income? How does it depend on the endogenous longevity mechanism? These are the questions I address below in this section. It is worth mentioning that [Straub \(2019\)](#) studies a related problem for the case in which both u and V are isoelastic. Indeed, under exogenous longevity and without health investment, the problem (1) reduces to the analysis Section 2.1 of [Straub \(2019\)](#). In contrast, in my setting the properties of $V(\omega)$ are not imposed exogenously but emerge endogenously from the second-stage portfolio choice.

Portfolio Problem. The second stage is a “portfolio” problem determining how total resources available at the beginning of second period ω are allocated between financial and health investment. Formally, given ω , the individual solves

$$V(\omega) \equiv \max_{\alpha, \lambda} \Psi(\lambda) \cdot u(R\alpha) \quad \text{subject to: } \alpha + \lambda = \omega. \quad (4)$$

The objects of interest here are the policy function for financial investment, $\alpha(\omega)$, and the associated value function, $V(\omega)$. In the absence of borrowing constraints, the optimal allocation is characterized by the equalization of returns across the two types of investment. The marginal return to financial investment is given by $R\Psi u'$, while the marginal return to health investment is given by $\Psi' u$. Rearranging this optimality condition and using the resource constraint, the optimal financial investment $\alpha(\omega)$ satisfies the following condition,

$$R \cdot \vartheta_u(R\alpha(\omega)) = \vartheta_\Psi(\omega - \alpha(\omega)) \quad (5)$$

where $\vartheta_u(c) \equiv \frac{u'(c)}{u(c)}$ and $\vartheta_\Psi(h) \equiv \frac{\Psi'(h)}{\Psi(h)}$ are the semi-elasticities of utility U and survival function Ψ , respectively. Given this policy function, the value function is thus defined as $V(\omega) = \Psi(\omega - \alpha(\omega)) \cdot u(R\alpha(\omega))$.

What are the properties of the financial investment function? Is it convex or concave as a function of initial wealth? How does it depend on the endogenous longevity mechanism? How does endogenous longevity shape the continuation value defined in (4)? Although [Hall and Jones \(2007\)](#) address some of these questions, I provide a more analytical and general characterization of the curvature of $\alpha(\omega)$. In particular, I formalize the concept of satiation which is the key object determining the curvature properties of both $\alpha(\omega)$ and $V(\omega)$.

Satiation Indices. I analyze how endogenous longevity affects the curvature of $a(y) \equiv \alpha(\omega(y))$ and its sensitivity to shifts in interest rate R and the survival function Ψ . This characterization requires the formalization of a concept of satiation rate for a generic function.

Definition 1. Take any continuously differentiable functions F and f such that $F, f : \mathbb{R} \rightarrow \mathbb{R}$ and let $\eta_f(x) \equiv \frac{f'(x)x}{f(x)}$ be the elasticity of F . The absolute satiation index of F , S_F , is defined as the elasticity

of F' relative to the elasticity of F'' , i.e., $S_F(x) \equiv \frac{\eta_{F'}(x)}{\eta_{F''}(x)}$. Similarly, the relative satiation index of F at x , S_F^{rel} , is the elasticity of $\vartheta_F \equiv \frac{F'}{F}$ relative to the elasticity of ϑ'_F , i.e., $S_F^{\text{rel}}(x) \equiv \frac{\eta_{\vartheta_F}(x)}{\eta_{\vartheta'_F}(x)}$.

The idea of satiation is formalized by the maximum of a concave function F , that is, the point at which $F' = 0$. The elasticity $\eta_{F'}$ captures the speed at which F' “approaches zero,” namely how quickly the level of F satiates. By contrast, $\eta_{F''}$ measures how fast this speed of satiation itself declines, that is, the rate at which the marginal returns flatten out. Consequently, higher values of $\eta_{F'}$ and lower values of $\eta_{F''}$ correspond to a higher degree of satiation of the function F . This motivates the definition of the absolute satiation index S_F as a natural summary measure of satiation.

The relative satiation index follows a similar logic, but instead of focusing on the speed and deceleration of F' , it applies the same concepts to the normalized marginal value $\vartheta_F = F'/F$. I now provide examples of functional forms for which these indices can be characterized analytically. I describe only their key properties here and relegate the analytical proofs to Appendix B.1. These properties are useful for interpreting the numerical illustrations presented in the analysis below.

Example 1: Isoelastic Function. Consider the function $F(x) = \underline{F}x^\xi$ with $\underline{F} > 0$ and $\xi \in (0, 1)$. The absolute and relative satiation index for F are constant and respectively given by $S_F = \frac{1-\xi}{2-\xi}$ and $S_F^{\text{rel}} = 1/2$. \square

Example 2: CRRA-Plus-Constant Function. Consider now the CRRA function added with a positive constant:

$$u(c) = \bar{u} + \frac{c^{1-\Sigma}}{1-\Sigma} \quad \text{with } \Sigma > 1 \quad (6)$$

for $c > c_{\min} \equiv [\bar{u}(\Sigma - 1)]^{\frac{1}{1-\Sigma}}$ and $\bar{u} > 0$ is a constant that guarantees the positiveness of u . Under this functional form, the absolute satiation index is constant, $S_F = \frac{\Sigma}{\Sigma+1}$. Moreover, the relative satiation index $S_F^{\text{rel}}(c)$ is non-linear, but it generally rises with consumption taking value 1/2 when consumption is near c_{\min} and it gradually converges towards $\frac{\Sigma}{\Sigma+1} > 1/2$ as consumption becomes very large. This parameterization obeys a common assumption in the literature following [Hall and Jones \(2007\)](#). \square

Example 3: Logistic Function. Now consider the following functional form:

$$\Psi(h) = \text{logistic}\left(\bar{\psi} + \bar{\psi}_h h^\xi\right) \quad \text{with } \bar{\psi}_h, \bar{\psi} \geq 0 \text{ and } \xi \in (0, 1) \quad (7)$$

where $\text{logistic}(x) = \frac{1}{1+e^{-x}}$ is the standard logistic function. This survival function Ψ satisfies the desirable properties: it is bounded between zero and one, satisfies Inada conditions with respect to health, and is consistent with estimated mortality models. The relative satiation index $S_\Psi^{\text{rel}}(h)$ increases with health investment, approaching $\frac{1-\xi}{2-\xi} < 1/2$ as h becomes very small, and it gradually rises toward 1 as h becomes very large. \square

3.1.1 On the Curvature of the Asset Function

In this part, I characterize the concavity of the asset policy $a(y)$. To do so formally, I define the concavity of a function f as $\sigma_f(x) \equiv -\eta_{f'}(x)$.⁸ I pay particular attention to how the endogenous longevity channel affects this curvature. Using the identity $a(y) \equiv \alpha(\omega(y))$, we can decompose the sources of concavity of a :

$$\sigma_a(y) = \sigma_\omega(\omega(y)) + \sigma_\alpha(a(y)) \cdot \frac{y}{\omega(y)}. \quad (8)$$

The concavity of $a(\cdot)$ therefore depends on the relative contributions of the concavity of $\alpha(\cdot)$ and $\omega(\cdot)$. I study both objects but abstract from lengthy algebraic derivations, focusing instead on the economic intuition and graphical analysis. All analytical proofs are provided in Appendix B.2. The key takeaway from my analysis is that $a(y)$ is concave for low incomes and convex for high incomes.

Benchmark Setting. As a benchmark, consider the case where longevity is exogenous, i.e., $\Psi(h) = \Psi \in (0, 1]$ for all h . In this case, the optimal investment in health is zero and the asset policy is given by

$$a^b(y) = \frac{1}{1 + \Psi^{-\frac{1}{\Sigma}} R^{1-\frac{1}{\Sigma}}} \cdot y. \quad (9)$$

Thus, when health investments do not affect longevity, the asset policy is linear in income level.

Intertemporal Problem. Consider the problem (2) and study the curvature of $\omega(y)$. The key result is that the concavity of the asset policy function depends on the difference between the absolute satiation rates for u and V . Consequently, a central aspect of the endogenous longevity mechanism is how it alters the satiation rate of the continuation function V .

Lemma 1. *The wealth policy function solving (2) is convex if and only if the absolute satiation index of u is higher than that of V , i.e., $\sigma_\omega < 0$, if and only if $S_u > S_V$.*

The relevance of Lemma 1 is that it identifies a unique condition for the convexity of the wealth policy function. The intuition for this condition is straightforward. When current consumption satiates faster than future resources, the marginal value of shifting consumption toward the second period declines more slowly than that of first-period consumption. Consequently, individuals allocate additional resources disproportionately toward the future, increasing saving at an accelerating rate. This mechanism generates convexity in the asset-income profile.

Straub (2019) studies a closely related problem in which preferences take the form $u(c) = \frac{c^{1-\Sigma}}{1-\Sigma}$ and $V(\omega) = \frac{\omega^{1-\hat{\Sigma}}}{1-\hat{\Sigma}}$. In that setting, the condition in Lemma 1 reduces to $\hat{\Sigma} < \Sigma$, which is the restriction emphasized in **Straub (2019)** as generating concavity in the consumption function, that is, preferences over future consumption are effectively more linear than preferences over current consumption.

⁸Mathematically, σ_f is the Arrow-Pratt measure of risk aversion and $\rho_f \equiv -\eta_{f''}$ is a measure of relative prudence, both frequently used in microeconomic models of choice under uncertainty. Although there is no uncertainty in this economy, these measures matter for the characterization below because they are, in general, indices of the curvature of F .

While this provides a convenient parametric condition for convexity of the wealth policy, it is not directly applicable in my framework. When longevity is endogenous, the continuation function V is an endogenous object. As a result, comparing only the coefficients of relative risk aversion of u and V is insufficient to determine the curvature of ω ; instead, the relevant statistic is the satiation index.

Below, I carefully examine how the endogenous longevity mechanism affects the satiation rate of V . While endogenous longevity can, in principle, generate a satiation index for V that is lower than that of u , the empirically and economically relevant case is the one specified in Lemma 1.

Portfolio Problem. I now characterize how Ψ affects the satiation of the continuation function V and the concavity of the policy function α . I show that the satiation index of V is generally lower than that of u , reflecting the fact that resources are more valuable when longevity is endogenous than when it is fixed. Moreover, unless the resource level ω is sufficiently large, the policy function α is concave, because consumption goods typically satiate faster than health goods.

Satiation Index of the Continuation Function. In the absence of health investments, the satiation index of V coincides with that of u , that is, $S_V = S_u$, as in the standard case. However, when individuals can invest in health, the satiation of V declines. Intuitively, because individuals can partially control their life expectancy through health investment, additional resources allow them to raise the value of future life. As a result, the diminishing returns to higher resources are less pronounced than under exogenous longevity.

Given the parameterizations (6) and (7), we can show that S_V is lower than S_u for low levels of ω , but eventually surpasses S_u once ω becomes sufficiently large.

Proposition 1. *Suppose that the functions u and Ψ are given by (6) and (7). The satiation index of the continuation function V solving (4) is lower than S_u except when wealth ω becomes sufficiently large. As a result, the wealth policy ω solving (2) is convex for all but sufficiently high levels of income y .*

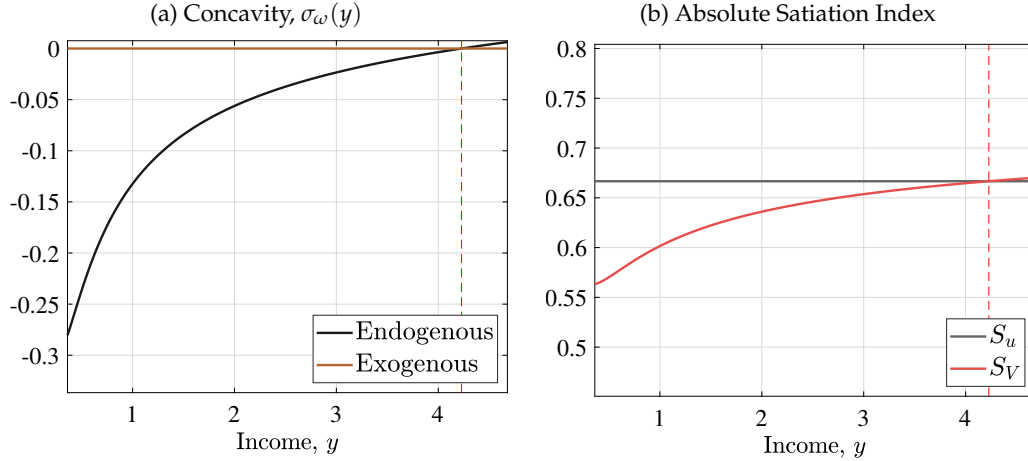
Panel (a) of Figure 3 shows that the investment policy $\omega(y)$ is convex under endogenous longevity. The source of this convexity is illustrated in panel (b): the absolute satiation index of the value function, S_V , is lower than that of u for low and middle income levels. This lower satiation rate means that additional resources have a higher marginal value when longevity is endogenous, because individuals can use part of those resources to improve survival prospects. As a result, saving responds more than proportionally to income at these levels, producing the convexity of $\omega(y)$. That is, endogenous longevity raises the marginal value of future consumption, shifting the allocation of resources toward old age.⁹

Concavity of the Financial Investment Function. I now study the concavity of α , σ_α , and show that it depends primarily on the relative satiation indices. This provides a framework to study the effects of Ψ on the curvature of the asset policy.

Proposition 2. *The financial investment policy solving (4) is concave if and only if the relative satiation index of u is higher than that of Ψ , i.e., $\sigma_\alpha > 0$, if and only if $S_u^{rel} > S_\Psi^{rel}$.*

⁹Although there exists a region in which $\omega(y)$ becomes concave, I abstract from it because it does not arise in my quantitative analysis and is not empirically relevant.

Figure 3: Intertemporal Problem and Total Wealth Policy



When the satiation rate of consumption, S_u^{rel} , exceeds that of health, S_V^{rel} , individuals reach diminishing marginal returns from additional consumption faster than from improvements in survival. In this case, health acts as a “luxury good” that continues to yield sizable utility gains relative to additional savings.¹⁰ Thus, as resources increase, households devote a larger share of those resources to health investment and a smaller share to financial saving, implying a declining marginal propensity to save.

Proposition 3. *Suppose parameterization given by (6) and (7). The relative satiation index for u is higher than S_V^{rel} except when wealth ω becomes sufficiently large. As a result, the financial investment policy α solving (2) is concave for all but sufficiently high levels of income ω .*

Proposition 3 also hinges on the fact that, under parameterization (6) and (7), consumption initially satiates faster than health: its relative satiation index starts at $1/2$, while longevity’s begins lower. This makes health relatively attractive at low resources, producing concavity in the optimal savings rule. But longevity’s satiation eventually overtakes consumption’s, causing health’s marginal value to collapse. At that point, saving becomes the dominant margin, and the policy becomes convex. This yields a threshold $\hat{\omega}$ where $\alpha(\omega)$ switches from concave to convex.¹¹

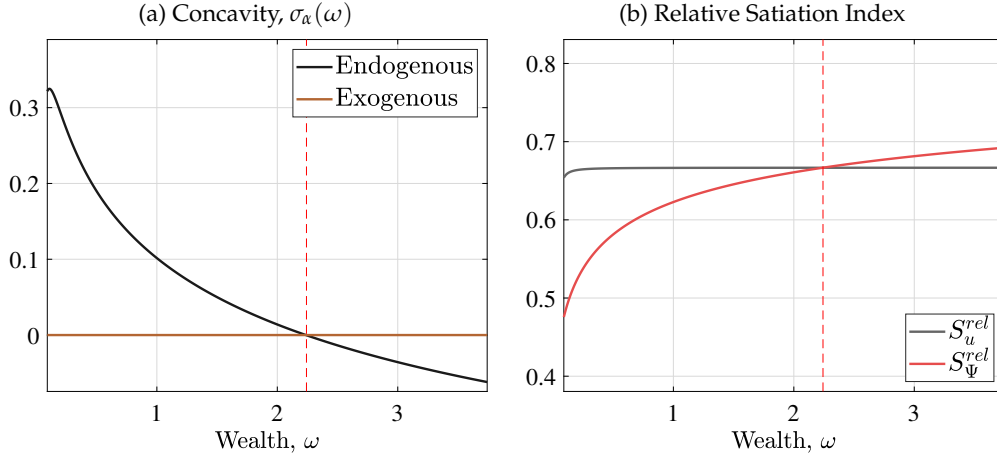
Figure 4 illustrates Proposition 3: α is concave for low to middle levels of resources. When resources become sufficiently large, the satiation of health investment exceeds the satiation of consumption, since additional health spending yields negligible longevity gains. As a result, $\alpha(\omega)$ eventually becomes convex.

Overall Asset Policy, $a(y)$. All in all, I have shown that the natural case is that financial and health investments compete for some given level of resources ω in (4). Whenever satiation in non-health goods is higher, then health investments are a luxury good so that financial investment policy is a

¹⁰As noted above, this intuition is closely aligned with the reasoning in Hall and Jones (2007) regarding the relationship between saving rates and income. What I do in this static subproblem is formalize that intuition, since the strength of this concavity will play an important role in determining the overall curvature of the asset policy $a(y)$.

¹¹Hall and Jones (2007) show that the concave region for α is the empirically relevant for aggregate measures. I do not ignore this region because income inequality across individuals is substantially larger than inequality across countries or states. As a result, reaching the convex region of α is more likely in my setting of interest.

Figure 4: Portfolio Problem and Financial Investment (4)



concave function with respect to total resources ω . Moreover, due endogenous longevity, the value function V becomes less satiable than U , inducing convexity in the optimal wealth policy that solves problem (2).

Due to endogenous longevity, the curvature of $a(y) \equiv \alpha(\omega(y))$ is pinned down by the relative importance of the concavity of α and ω as shown in (8). It turns out that, when income is low, the asset policy is concave as the satiation of consumption in the second period is relatively fast and individuals invest more in health to take advantage of the large longevity gains due to health investments. Instead, as income increases the longevity gains are relatively low so that the agent start investing more in assets rather than in health.

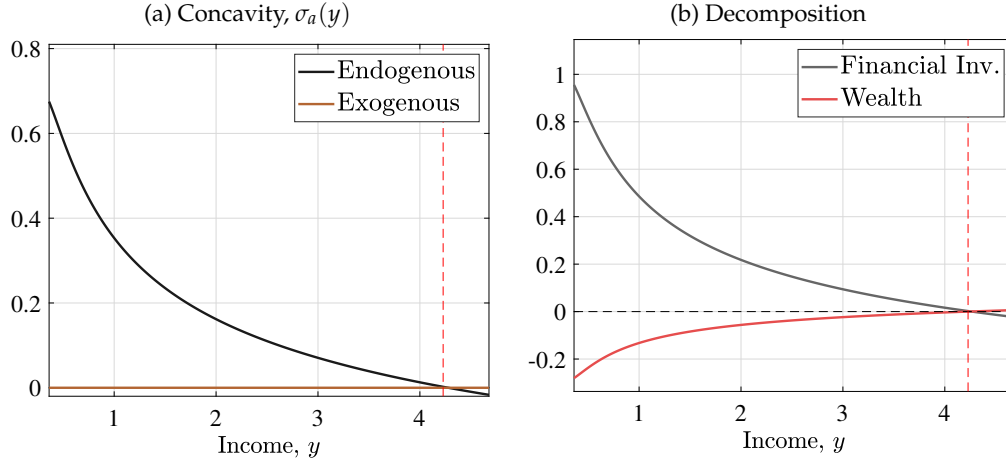
Proposition 4. *Suppose parameterization given by (6) and (7). It turns out that there exists a threshold \hat{y} such that the asset policy $a(y)$ solving problem (1) is concave for $y \in (0, \hat{y}]$ and convex otherwise.*

Figure 5 summarizes the properties of the overall asset policy $a(y) = \alpha(\omega(y))$ based on Proposition 4. Panel (a) shows that asset accumulation under endogenous longevity lies below the exogenous case for all but very low income levels, reflecting the fact that part of resources is diverted toward health investment. Despite this downward shift, panel (b) shows that the asset policy remains globally increasing in income and exhibits a characteristic change in curvature: it is concave at low income levels and becomes convex once income is sufficiently high. The source of this curvature pattern is illustrated in panel (c). For low income, the absolute satiation index of the value function, S_V , is below that of U , implying that future resources have a high marginal value because they increase both consumption and survival.

3.1.2 Elasticities of Savings

Another aspect altered by the endogenous longevity channel on the individual decisions is the elasticity of individual savings with respect to other aspects of their circumstances. In particular, I am interest is in movements in interest rate and survival rate parameters. In a standard setting with

Figure 5: The Asset Policy Function



exogenous longevity, higher life expectancy or a higher real interest rate mechanically increases asset demand, as agents plan for longer lifespans or intertemporal substitution of consumption.

When longevity is instead an outcome of health investment, the response of saving depends on how such changes alter the incentives to invest in health. Let x denote an exogenous movement in either interest rate or a parameter in the survival rate function, then the elasticity of savings with respect to x is

$$\frac{\partial a}{\partial x} \frac{x}{a} = \frac{\partial \alpha}{\partial x} \frac{x}{\alpha} + \eta_\alpha \cdot \frac{\partial \omega}{\partial x} \frac{x}{\omega} \quad \text{where} \quad \frac{\partial \omega}{\partial x} \frac{x}{\omega} \equiv \frac{\frac{\partial V'}{\partial x} \frac{x}{V'}}{\frac{\omega}{c_y} \sigma_u + \sigma_V}.$$

Using these formula for exogenous movements in survival technology we have

$$\frac{\partial a}{\partial x} \frac{x}{a} = -\frac{\frac{\partial \theta_\Psi}{\partial x} \frac{x}{\theta_\Psi}}{\eta_U + \frac{\alpha}{\lambda} \eta_\Psi} + \eta_\alpha \cdot \frac{\frac{\partial \Psi}{\partial x} \frac{x}{\Psi}}{\frac{\omega}{c_y} \sigma_u + \sigma_V}.$$

If an exogenous improvement in survival technology reduces the marginal return to health spending (e.g., because these advances allow individuals to live longer with the same spending) agents reallocate resources from health to financial assets. In this case, the responsiveness of saving to longer lifespans is amplified, as individuals reinforce the direct effect of higher survival by saving more. Conversely, if the same improvement increases the productivity of health investment (i.e., making additional spending on health more effective) agents substitute toward health, and the saving response is dampened.

A similar reasoning applies to changes in the interest rate: when longevity gains from health investment are large, higher returns to saving mainly raise asset accumulation; when these gains are limited, part of the additional resources is diverted toward health, reducing the elasticity of saving to interest rate movements. This can be seen in the following formula:

$$\frac{\partial a}{\partial R} \frac{R}{a} = \frac{1 - |\eta_{\theta_U}|}{|\eta_{\theta_U}| + \frac{\alpha}{\lambda} |\eta_{\theta_\Psi}|} + \eta_\alpha \cdot \frac{1 - \sigma_U}{\frac{\omega}{c_y} \sigma_u + \sigma_V}.$$

Overall, compared to a framework with exogenous longevity, the model implies changes in the sensitivity of individual savings to demographic or interest rate: Saving responds more strongly when the reallocation toward financial assets dominates (crowding-in case) and more weakly when health investment absorbs a larger share of income (crowding-out case).

3.2 Aggregate Savings and General Equilibrium

I embed the preceding microeconomic environment into a neoclassical overlapping-generations framework where time is discrete and extends to infinity. The resulting model extends [Diamond \(1965\)](#) by introducing endogenous longevity.

3.2.1 Laissez-Faire Equilibrium

Demographics. At any date t , the number of youngs and olds are denoted by $N_{y,t}$ and $N_{o,t}$, whereas the total population is $N_t \equiv N_{y,t} + N_{o,t}$. On average, youngs at time t are alive at time $t + 1$ with probability ψ_{t+1} . Thus, given a sequence of fertility rates $\{n_t\}_{t=0}^{\infty}$ and survival probabilities $\{\psi_t\}_{t=0}^{\infty}$, the total number of youngs (new borns) and olds are $N_{y,t} = (1 + n_t)N_{y,t-1}$ and $N_{o,t} = \psi_t N_{y,t-1}$, respectively. Thus, the dependency ratio is simply the ratio of the number of old and young individuals:

$$d_t \equiv \frac{N_{o,t}}{N_{y,t}} = \frac{\psi_t}{1 + n_t} \quad (10)$$

and the population growth rate is $1 + n_t^{\text{pop}} = \frac{1+n_t+\psi_t}{1+d_{t-1}}$. Although fertility rate is exogenous in my model, the demographic structure is not as both the dependency ratio and population growth are the result of individual health investment decisions.

Health technology. In this economy, the survival function Ψ_t is allowed to vary over time. Shifts in Ψ_t capture exogenous improvements in life expectancy driven by medical progress, such as advances in cancer treatment.

Production Technology and Firms. The economy has access to a constant-return to scale neoclassical technology which combines physical capital and labor to produce final goods: $Y_t = F(K, ZL)$ where Z is a labor-productivity parameter. Moreover, capital depreciates at a constant rate $\delta^k \in (0, 1)$. There is a representative firm operating this neoclassical technology in a perfectly competitive market for final goods. The representative firm chooses capital and labor force, K and L , to maximize profits $F_t(K, L) - r_t^k K - w_t L$, leading to the following optimality conditions:

$$r_t^k = \partial_K F_t(K, L) \quad \text{and} \quad w_t = \partial_L F_t(K, L) \quad (11)$$

where r_t^k is the capital net return and w_t is the wage rate.

Financial Markets. In the laissez-faire economy, there exists two riskless assets: financial contracts and physical capital. Since asset markets are frictionless, the return of these assets satisfy the non-arbitrage condition $R_t = 1 - \delta^k + r_t^k$ where r_t^k is the return on capital and R_t is the return of financial contracts.

As agents face longevity risk but do not have access to annuities, bequests are accidental which are distributed uniformly among new borns. As assets a_t are bequestable, the bequests receive for new borns at period t is

$$b_{q,t} \cdot N_{y,t} = R_t \cdot N_{y,t-1} \left\{ (1 - \bar{\psi}_t) \cdot \bar{a}_{t-1} \cdot \bar{\pi} + (1 - \underline{\psi}_t) \cdot \underline{a}_{t-1} \cdot \underline{\pi} \right\} \quad (12)$$

where \bar{x} and \underline{x} denote choices of individuals with productivities \bar{z} and \underline{z} , respectively.

Labor-Productivity Types and Individual Behavior. Each individual borns with labor productivity z_t and supply their unitary labor endowment inelastically. Labor productivity may take two values, \underline{z}_t with probability $\underline{\pi}$ and value \bar{z}_t with probability $\bar{\pi}$. The average of labor productivity is normalized to be one. Households' problem is essentially the same as in Section 3.1 with time-varying interest rate, survival function, and income. In fact, the budget constraints are

$$c_{y,t} + h_t + a_t = w_t \cdot z + b_{q,t} \quad (13)$$

$$c_{o,t+1} = R_{t+1} \cdot a_t. \quad (14)$$

Saving decisions obey the standard Euler equation but with an expected return is affected by their survival rates:

$$u'(c_{y,t}) = R_{t+1} \cdot \Psi_{t+1}(h_t) \cdot u'(c_{o,t+1}). \quad (15)$$

The optimal health investment balances the current marginal cost of higher health expenses with the marginal benefits of health expenses, in turn, determined by the value of future life and the changes in medical expenses. We can write this optimality condition as equalizing the return on financial assets and health stock as follows

$$R_{t+1} = \frac{\Psi'_{t+1}(h_t)}{\Psi_{t+1}(h_t)} \cdot \frac{u(c_{o,t+1})}{u'(c_{o,t+1})}. \quad (16)$$

Equilibrium. The laissez-faire equilibrium is a set of allocation paths $\{h_t, c_{y,t}, c_{o,t}, k_{t+1}\}$ satisfying conditions (13)-(16) together with (11), (12), and the market clearing conditions $K_{t+1} = N_{y,t} \sum_z a_t(z) \pi_t^z$ and $N_{y,t} = L_t$, where $a_t(z)$ are the saving decisions for z -type individuals given price paths $\{R_t, w_t\}$.

For pedagogical reasons, I rewrite the equilibrium definition such that I can focus on only one object: savings decisions as a function of income (asset function). The gross return and wage rates are functions of the capital-labor ratio, $k_t = K_t/N_{y,t}$, which in this model is independent of health investment decisions. As bequests and prices depend on the (past) capital-labor ratios, we can write the optimal asset decisions for an individual with productivity z as $a_t(z; k_{t+1}, \Psi_{t+1})$. Given the individuals' asset functions, the aggregate asset policy is

$$\mathcal{A}_t(k_{t+1}, \Psi_{t+1}, \Pi_t^z) \equiv a_t(\bar{z}_t; k_{t+1}, \Psi_{t+1}) \cdot \bar{\pi} + a_t(\underline{z}_t; k_{t+1}, \Psi_{t+1}) \cdot \underline{\pi}. \quad (17)$$

Using this definition, I can re-defined the laissez-faire equilibrium as follows.

Definition 2. Given an initial stock of capital k_0 and exogenous movements for inequality and demographics $\{\Pi_t^z, \Psi_t, n_t\}_{t=0}^\infty$, the equilibrium path for the capital stock is given by

$$\mathcal{A}_t(k_{t+1}, \Psi_{t+1}, \Pi_t^z) = (1 + n_{t+1}) \cdot k_{t+1} \text{ for all } t \geq 0. \quad (18)$$

where $\mathcal{A}_t(\cdot)$ is defined as in (17).

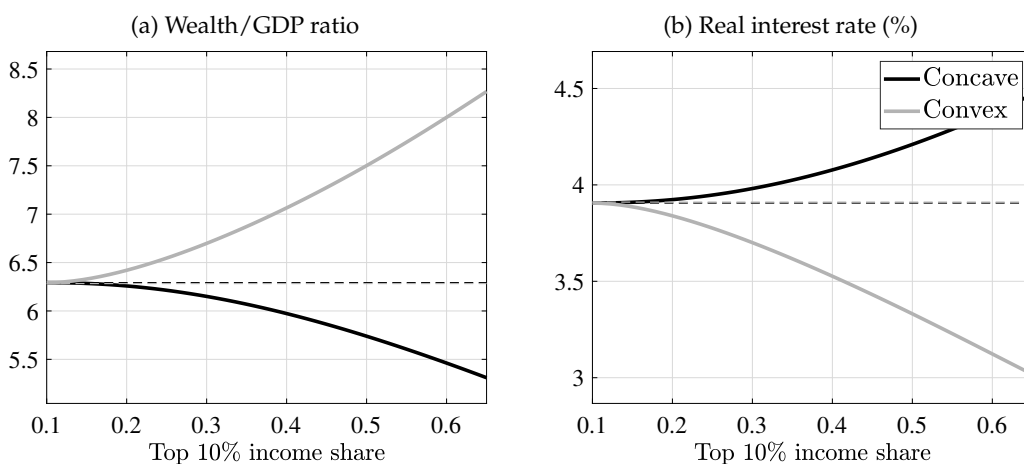
Based on (18), to study the partial and general equilibrium effects of inequality and demographics, it is enough to characterize the sensitivity of individual policy to productivity (or income), to exogenous changes in survival rates, and to interest rates (or, more generally, stock of capital). I pay special attention to disentangle the role of the endogenous health formation.

3.2.2 General Equilibrium Effects of Inequality

So far, I have explored the consequences of endogenous longevity over individual savings. I consider now the aggregate consequences of endogenous longevity, both at partial and general equilibrium level. In this section, I focus on the effects of inequality, but the same analysis for exogenous movements in survival and fertility rates are in Appendix B.4.

Exogenous inequality shifts are captured by changes in \underline{z}_t and \bar{z}_t satisfying the unitary mean. In particular, I measure inequality by the share of aggregate (before-tax) labor income accrued by high-productivity individuals, $\bar{z}_t \pi$. From the clearing market condition (18), our previous characterization of the curvature for assets is to determine the exogenous partial equilibrium movements. In particular, if concavity is predominant in the asset policy, rises in inequality will drive aggregate savings down and real interest rates up. Instead, when the convexity is predominant, increments in inequality drives the aggregate savings up and the real interest rates down. Figure 6 illustrates the general equilibrium effects for aggregate savings and interest rates at the steady state for different levels of the income share of the top 10% in the income distribution.

Figure 6: Equilibrium Effects of Increment in Income Inequality



Note. Solid lines represent the model with endogenous longevity, while dashed lines model with exogenous longevity.

3.3 Discussion

Now, I discuss the effects of relaxing some of the assumptions I made in the previous analysis.

Curative Medical Expenses. In Section 3.1, I have abstracted from medical expenses for elderly, M . When these medical expenses are present (e.g., long-term care needs) there is an extra reason for having a greater concavity in α and lower satiation V . Intuitively, if health investment reduces non-utility expenditure in the future, then health is more desirable, generating more desirability of ω and health as a share of it. As a result, medical expenses increases the threshold income \hat{y} at which the asset policy turns convex.

Labor Productivity and Utility as Function of Health. The presence of labor productivity as a function of health generates the same effects as the introduction of medical expenses $M(h)$. Certainly, the function M may capture the net effect of any other resource that affects the individual's budget constraint and depends on health.

Although health dependence of utility do not enter the budget constraint as medical expenses, the effects of these model ingredients are quite similar. In fact, having the interaction of survival rate Ψ and utility U in the model explored in Section 3.1 suggests that making U a function of health too will only reinforce the reduction in satiation of the value function V and, then, reinforce the concavity region for the asset policy.

Bequest Motives. So far I have assumed that individuals do not derive utility from giving bequests to the future generations. In reality, some of the intergenerational transfers are not accidental but voluntary. Having bequest motives can certainly affect my results. However, as long as utility from bequests are not greater than utility from consumption, the previous results go through but will be less quantitatively appealing. The role of bequest motives will be more important for the normative analysis described below.

Annuity Markets. In Appendix B.2, I show that the results go through in the presence of annuity markets.

4 First- and Second-Best Allocations

This section examines how an economy with endogenous longevity can achieve efficient outcomes through optimal policy design. Before analyzing realistic institutional constraints, I first characterize the first-best allocation, where a social planner maximizes welfare subject only to technological and resource constraints. I then interpret its optimality conditions and identify the fiscal and redistributive instruments that could replicate it. Next, I move to the second-best environment, where the government faces informational and market limitations, to study how these imperfections reshape optimal health and pension policy.

4.1 First-Best Allocation

Before discussing the optimal design of pension and health system under realistic assumptions, I explore the allocations that are part of the Pareto frontier. Comparing how these allocations are determined with the competitive allocations will be a first step towards understanding the source of inefficiencies in the economy and how a government can restore efficiency in the competitive equilibrium. In this section, I explore the resulting first-best allocation, while second-best allocations are studied in the next section.

4.1.1 Planning Problem

A social planner distributes resources subject to technological constraints. This planner assigns individual bundles to each individual to maximize a given social welfare function subject to aggregate constraints of the economy.¹² Suppose that the Pareto weights of the planner are $\{\Phi_{t-1}\}_{t \geq 0}$ satisfying $\sum_{t=0}^{\infty} \Phi_t < \infty$ for all $t \geq 0$. Given K_0 and $\{\mathcal{N}_{t-1}\}_{t \geq 0}$, the planner chooses $\{c_{y,t}^i, c_{o,t}^i, h_t^i\}_i, K_{t+1}\}_{t \geq 0}$ to maximize

$$\sum_{t=0}^{\infty} \frac{\Phi_t}{N_{y,t}} \int_{i \in \mathcal{N}_t} \left[u(c_{y,t}^i) + \Psi_{t+1}(h_t^i) u(c_{o,t+1}^i) \right] \partial i + \frac{\Phi_{-1}}{N_{y,-1}} \int_{i \in \mathcal{N}_{-1}} \Psi_0(h_{-1}^i) u(c_{o,0}^i) \partial i$$

subject to the resource constraint

$$F_t(K_t, N_{y,t}) + (1 - \delta)K_t = \int_{i \in \mathcal{N}_t} (c_{y,t}^i + h_t^i) \partial i + \int_{i \in \mathcal{N}_{t-1}} \Psi_t(h_{t-1}^i) c_{o,t}^i \partial i + K_{t+1}$$

where \mathcal{N}_t is the set of individuals born at date t . Note that I impose uniform Pareto weights within generation's individuals so that this is a utilitarian planner. Under concavity in u , first-order conditions are sufficient to characterize optimal paths in the standard settings. With the current endogenous-health setting, however, additional restrictions are required.

Assumption 1. *The value of life as a ratio to consumption is larger than one, $\frac{U(c)}{U'(c)c} \geq 1$, and past investment decisions are uniform: $h_{-1}^i = h_{-1}$ for all $i \in \mathcal{N}_{-1}$.*

The assumption that $\frac{U(c)}{U'(c)c} \geq 1$ indicates that the value of life rises faster than consumption. A natural way to interpret this is that there are unmodeled reasons that make the second-period worth living.¹³ Under this assumption, the social planner distributes resources uniformly for all agents within generations. This assumption allows me to omit inequality changes and focus on the first-best allocation in response to healthcare technology shocks (captured by shifts in Ψ).¹⁴ Thus, the planning

¹²Since I take an exogenous stream of newborns, the endogeneity of the population size comes from the survival risk individuals face at the second period, the veil of ignorance fiction can be used in favor of a utilitarian social welfare function. That is, I do not face normative complications to use this social welfare function as other works have emphasized (e.g., [de la Croix and Doepke \(2021\)](#)).

¹³This type of assumptions is standard in the literature studying endogenous health investments or mortality rate.

¹⁴I return to the role of inequality increments when studying the second-best allocations in Section 4.3.

problem can be written as follows: Given the initial stock of capital k_0 and a path for fertility and healthcare technology $\{\Psi_t\}_{t \geq 0}$, the planner chooses $\{h_t, c_{y,t}, c_{o,t}, k_{t+1}\}_{t \geq 0}$ to maximize

$$\sum_{t=0}^{\infty} \Phi_t [u(c_{y,t}) + \Psi_{t+1}(h_t)u(c_{o,t+1})] + \Phi_{-1}\Psi_0(h_{-1})u(c_{o,0}) \quad (19)$$

subject to the resource constraint

$$f_t(k_t) + (1 - \delta)k_t = c_{y,t} + h_t + \frac{\Psi_t(h_{t-1})}{1 + n_t}c_{o,t} + (1 + n_{t+1})k_{t+1}. \quad (20)$$

where $f_t(k) \equiv F_t(k, 1)$ and $k_t \equiv K_t/N_{y,t}$. Proposition 5 states the optimality conditions for the planner's problem, shedding lights on the sources of potential inefficiencies in a competitive equilibrium allocation.

Proposition 5. *Under Assumption 1, the planner allocates the same $(h_t, c_{y,t}, c_{o,t+1})$ for all agents in generation t and this allocation is characterized by: (i) optimal intragenerational allocation on consumption and health investments:*

$$u'(c_{y,t}) = (1 + \chi_{s,t+1}) R_{t+1} \Psi_{t+1}(h_t) u'(c_{o,t+1}) \quad (21)$$

$$(1 + \chi_{h,t+1}) (1 + \chi_{s,t+1}) R_{t+1} = \frac{\Psi'_{t+1}(h_t) u(c_{o,t+1})}{\Psi_{t+1}(h_t) u'(c_{o,t+1})} \quad (22)$$

where $\chi_{s,t+1} \equiv \frac{1}{\Psi_{t+1}} - 1 \geq 0$, $\chi_{h,t+1} \equiv \frac{\Psi'_{t+1}}{R_{t+1}} c_{o,t+1} > 0$ and $R_{t+1} \equiv 1 - \delta + f'_{t+1}(k_{t+1})$; and (ii) an optimal intergenerational allocation of resources $\frac{u'(c_{y,t})}{u'(c_{o,t})} = \phi_t^{-1}(1 + n_t)$ where $\phi_t \equiv \Phi_t/\Phi_{t-1}$ is the planner's relative concern of generation t with respect to generation $t - 1$.

According to Proposition 5, there are two inefficiencies/wedges in a competitive equilibrium that are related to the within-generation allocation.¹⁵ These inefficiencies in the competitive equilibrium provide the rationale for government intervention aimed at influencing individuals' decisions over physical and health capital. Such interventions take the form of taxes, subsidies, and financial market policies designed to correct distortions in each type of capital accumulation. Policies targeting physical capital are associated with the pension system, while those aimed at health capital are implemented through the medical system.

Saving Wedge, $\chi_{s,t+1}$. At the optimal allocation, the within-generation consumption path is determined by the marginal rate of intertemporal transformation, that is, the return on capital. The planner effectively pools survival risk across individuals, ensuring that consumption profiles are independent of individual mortality. In contrast, in the laissez-faire equilibrium, agents adjust their consumption based on their own survival probabilities. This inefficiency arises from incomplete markets, particularly the absence of a perfect annuity market to insure against longevity risk.

Health-Investment Wedge, $\chi_{h,t+1}$. Individuals may not internalize the impact of their health stock on the demographic structure; specifically, on the share of elderly individuals in the population. A higher

¹⁵There may be a third source of inefficiency which is related to the between generation allocation: Dynamic inefficiency caused by "shortage of assets" and/or overaccumulation of capital (see Samuelson (1958) and Diamond (1965)). Throughout this document, I suppose the economy is dynamically efficient.

health stock for generation t increases the number of old individuals alive in period $t + 1$, which creates a negative externality for generation $t + 1$. In particular, a larger elderly population imposes greater challenges for accumulating resources (i.e., the capital stock) needed to support future generations. Therefore, there is over-investment in health and underinvestment of physical capital.¹⁶

Samuelson (1975a) makes a similar point to this demographic externality. He argues that in a decentralized economy with endogenous fertility decisions, there may be an intergenerational misalignment as each generation benefits from higher population growth (e.g., due to old-age support), but no one internalizes the negative externalities imposed on future generations. This may lead to a non-optimal equilibrium, where society keeps expanding population unsustainably. The demographic externality I discuss in this paper is similar in spirit to this Samuelson's concern.

It is worth mentioning that similar "externalities" appear in settings where annuities are publicly and/or privately provided as long as they do not penalize health investments. In these settings, the health-investment wedge takes the form of a moral hazard problem and it was first explored by Davies and Kuhn (1992) and Becker and Philipson (1998).

4.1.2 Increments in the Survival Rate

Suppose the economy is initially at the stationary equilibrium associated with a survival function $\Psi_-(\cdot)$, and planner reoptimizes the allocation at date 0 when confronted with a shift in the survival function to $\Psi_+(\cdot)$ such that $\Psi_+(h) > \Psi_-(h)$ for all $h \in \mathbb{R}_+$. How is the evolution the optimal path $\{h_t, c_{y,t}, c_{o,t}, k_{t+1}\}$? I answer this question in two parts. I first explore the difference between the long-run planner's allocation relative to the initial allocation and, next, I study the dynamics of the first-best allocation.

Long-Run. Despite the broader debate on how longevity affects real interest rates, Proposition 5 establishes that, in the long run, the real interest rate is independent of the survival rate. Neither survival risk nor endogenous health stock influences the "golden rule" of capital accumulation as the interest rate is $\phi^{-1}(1+n) + \delta - 1$. Why is the capital-labor ratio constant? Without loss of generality suppose zero population growth (i.e., $n = 0$). The planner chooses the aggregate stock of capital K based on technological returns and intergenerational preferences, neither of which is affected by the survival rate. The aggregate capital stock remains constant after a shift in ψ and, as the size of the young individuals does not change, the stock of capital-labor ratio remains independent of Ψ .

Next, I examine the comparative statics of the planner's allocation with respect to a rise in the longevity parameter Ψ . As solving the general case analytically is intractable, I impose structure on preferences and functional forms to characterize the steady-state response.

Proposition 6. *Under Assumption 1, the steady level of the stock of capital is independent of the survival rate function. Moreover, under parameterization (6) and (7), health investment and consumption fall with $\bar{\psi}$. Instead,*

¹⁶It is worth emphasizing that, the agents' and planner's valuation of the other consequences of health investments (i.e., higher joy of life) are aligned.

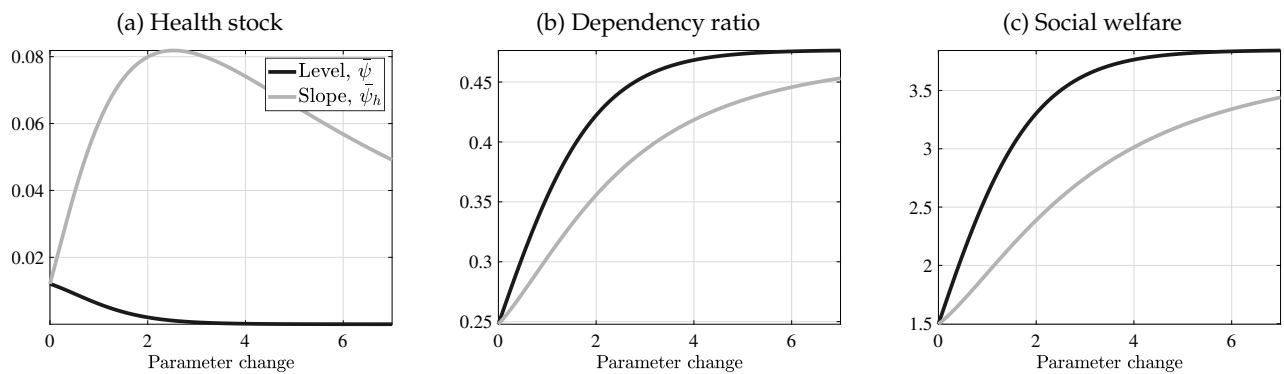
when $\bar{\psi}_h$ rises, consumptions fall but health investment rises (resp. declines) if $\bar{\psi}_h < \widehat{\psi}_h$ (resp. $\bar{\psi}_h > \widehat{\psi}_h$) where $\widehat{\psi}_h$ is defined in Appendix E.6.

This result hinges on a standard decomposition into substitution and income effects. An increase in $\bar{\psi}_h$ raises the marginal return to health investment, encouraging higher health accumulation (substitution effect). Simultaneously, it increases the proportion of elderly individuals, raising aggregate medical expenses and tightening the resource constraint (income effect). When the health stock is low, the income effect is muted because few individuals survive to old age and medical costs are low. As the health stock rises, however, the income effect dominates, eventually leading to a decline in health investment. Since the capital-labor ratio does not change, the decline in consumption for both young and old aged reflects a reallocation of resources away from consumption and toward health investment, while the aggregate consumption remains constant.¹⁷ For movements in $\bar{\psi}$, instead, generally these two effects go in the same direction and welfare increases unambiguously.

What are the welfare implications of a rise in $\bar{\psi}_h$? Interestingly, Proposition 6 suggest that welfare gains from longer life expectancy are non-monotonic. Initially, higher $\bar{\psi}_h$ improves welfare through better health and longevity. But beyond a threshold (potentially higher than $\widehat{\psi}_h$), the income effect is that strong that a rise in $\bar{\psi}_h$ reduces health investment and lower consumption, ultimately reducing welfare—even in the long run. We explore this possibility further in the numerical illustration below.

Figure 7 illustrates the results of Proposition 6.¹⁸ The optimal health stock and consumption exhibits a decline relationship with $\bar{\psi}$. Although health investment decline, life expectancy continues to rise as reflected by the increasing dependency ratio so that the change in survival rate is generally welfare increasing.

Figure 7: Stationary Planner’s Allocation as a Function of Survival Rate Parameters



Note. The x-axis labeled as “Parameter change” denotes the absolute increments in a general parameter ψ governing $\Psi(\psi)$.

Dynamics. I characterize the dynamics of this permanent change in the survival rate. Suppose the initial conditions are given by $k_0 = k_-$, $\mu_0 = \mu_-$, and $h_{-1} = h_-$ where (k_-, μ_-, h_-) corresponds to the stationary allocation under $\bar{\psi} = \bar{\psi}_-$ and $\bar{\psi}_h = \bar{\psi}_{h,-}$. Now, consider permanent increments in the

¹⁷This intuition relies on the assumption that health does not affect labor productivity. If, instead, health improvements raised worker productivity, then a rise in Ψ would lead to greater investment in both health and physical capital.

¹⁸Details about this and the next numerical exercise are in Appendix B.6.

parameters $\bar{\psi}$ and $\bar{\psi}_h$ and let us explore the dynamic response for k , h and c_0 . Proposition 7 summarizes these dynamic responses.¹⁹

Proposition 7. *Suppose Assumption 1 and parameterization (6) and (7) hold. Assume the social planner re-optimizes at the moment of this new longevity regime operates (i.e., at $t = 0$). If $\bar{\psi}_{h,+} > \bar{\psi}_-$ and $\bar{\psi}_+ = \bar{\psi}_-$, there is (i) an initial rise in the capital-labor ratio, $k_1 > k$, followed by gradual reversion toward its initial steady-state level; (ii) an overshooting response of the health stock, $h_0 > h_+$, with a subsequent monotonic decline toward the new, higher steady-state value h_+ ; and (iii) a declining path of consumption in both youth and old age converging toward their respective new, lower steady-state levels.*

The proposition implicitly reveals a dynamic redistribution mechanism: early generations (especially the elderly at $t = 0$) bear some welfare losses, while future generations reap the gains from improved health and longevity. This intertemporal redistribution poses a challenge for political feasibility and highlights the importance of transitional compensation mechanisms (e.g., targeted transfers) to maintain intergenerational equity. Importantly, the gains from longer life expectancy materialize only gradually, and in the case of a slow increase in the survival function, they are realized primarily by future generations. This intertemporal distribution of gains is a central aspect we further explore in the next numerical illustration.

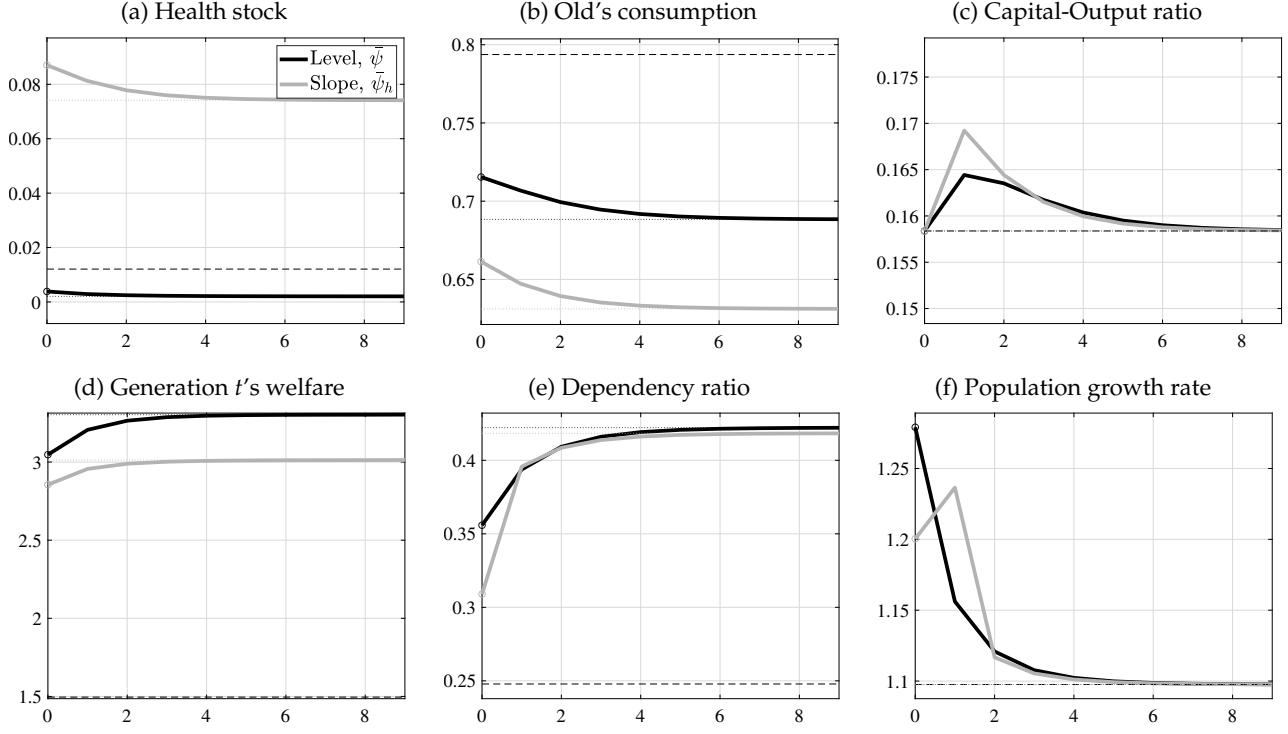
Under the assumption that substitution effects outweigh income effects, an exogenous improvement in survival technology enhances the welfare of future generations by enabling them to live longer, healthier lives even if it requires sacrificing some consumption. What is the optimal transition toward this more desirable steady state? Figure 8 illustrates this transition after a once-and-for-all change in survival technology. At date 0, the social planner anticipates a larger future population and responds by increasing savings in physical capital. At the same time, the higher marginal return to health investments induces a surge in health stock. These investments are financed through a reduction in old-age consumption. From date 1 onward, the economy begins to decumulate physical capital to sustain the growing population and continued health investments. Notably, the health stock overshoots at date 0 and gradually converges downward toward its new higher steady-state level. This front-loading of health investment—and the associated concentration of welfare gains early in the transition—is a hallmark of the first-best allocation. It reflects (i) the dominance of the lifetime substitution effect over the negative income effect of aging, and (ii) the planner’s prioritization of consumption smoothing for future generations at the expense of the current elderly.

4.2 Implementation of the First-Best Allocation

In general, the first-best allocation can be implemented through either private or public mechanisms. In this section, I explore how a “powerful” enough government can implement a first-best allocation. In particular, I assume that the government can use any of the instruments specified in the following assumption.

¹⁹We can easily illustrate the dynamics by using a phase-diagram style heruistics. In Appendix C.3, I show that the planner’s allocation can be written as two-equation system in the (c_0, k) –space very similar to the neoclassical phase diagram.

Figure 8: Planner's Allocation in Response to Increments in Life Expectancy



Note. Solid lines are the optimal path under Proposition 7 after a smooth increment in ψ ; black-dashed line represents the initial steady-state level; and the dotted line denotes the final steady-state level.

Assumption 2. The government linearly taxes (i) saving returns, τ_t^k , and (ii) health investments, τ_t^h . Assuming perfect information, the government collects labor income taxes $\{\bar{\tau}_t, \underline{\tau}_t\}$ from the young and provides transfers $\{\bar{s}_t, \underline{s}_t\}$ to elderly. It also offers annuities to insure against longevity risk. Any resulting primary deficit is financed through government debt, $B_{g,t}$.

4.2.1 Fair Annuities and Pigouvian Correction

This government can implement the social planner's allocation corresponding to a given set of Pareto weights $\{\phi_t\}$. To do so, it must redistribute resources within each generation, insure individuals against medical expense risk, complete the annuity market, and tax health investment to internalize the negative externality associated with health capital. Proposition 8 formalizes this result and characterizes the optimal set of government interventions under these conditions.

Proposition 8. A government as in Assumption 2 can implement the social planner allocation (with Pareto weights $\{\phi_t\}$) by providing: (i) within-generation perfect redistribution, $\tau_t(z) = (z - 1)w_t + \tau_t$, (ii) uniform pension, $s_t = s_t(z)$, (iii) fair annuity system with return $R_{t+1}^*/\Psi_{t+1}(h_t^*)$ without tax on savings, and (iv) a Pigouvian tax on health investment,

$$\tau_{h,t} = \chi_{h,t+1}^* = \frac{\Psi'_{t+1}(h_t^*)}{R_{t+1}^*} c_{o,t+1}^* > 0 \quad (23)$$

where x^* is the social planner's allocation for variable x . Public debt is zero and the intergenerational transfer (τ_t, s_t) makes the allocation consistent with Pareto weight $\{\phi_t\}$ and the government budget constraint, $\tau_t + \tau_t^h h_t^* = d_t^* s_t$.

The first two aspects of the optimal implementation does not necessarily require a government. All what is needed is to have complete financial markets in the sense that they provide fair annuities and fair medical insurance contracts. This may seem plausible as it all require to pool idiosyncratic risk in the proper way. It is more difficult to envision a market-based solution to address the negative externality associated with health investments. In this context, a Pigouvian tax on health investment emerges as a central policy tool for aligning decentralized decisions with the socially optimal allocation. Intuitively, the higher the health investments impact on the aggregate old's consumption, $\chi_{h,t+1}^*$, the higher the necessary correction.

Under this optimal policy implementation, I distinguish between a pension system and a health system, each comprising two key components. The pension system is responsible for (i) pooling longevity risk within each generation through actuarially fair annuities and (ii) facilitating intergenerational transfers to support the elderly. In turn, the health system addresses (i) the provision of universal access to curative medical care and (ii) the implementation of Pigouvian taxes on preventive health investments to correct for externalities associated with population aging.

These two systems interact in two critical ways. First, improvements in preventive health investments—guided by the health system—affect survival rates and thus alter the size of the risk pooled by the pension system and the size of beneficiaries of the pension system. Note that the promised return of the pension system has an individual-account component given by $R_t^* / \Psi_t(h_{t-1}^*)$. Second, the intergenerational transfers managed by the pension system must adjust to the medical expenditure needs and longevity patterns shaped by health policy. This interdependence highlights the importance of coordinated design in achieving an efficient and equitable allocation. Specifically, note that the social security benefit has a lump-sum component given by

$$\bar{s}_t = c_t^{o,*} - \frac{R_t^*}{\Psi_t(h_{t-1}^*)} (1 + n_t) k_t^*.$$

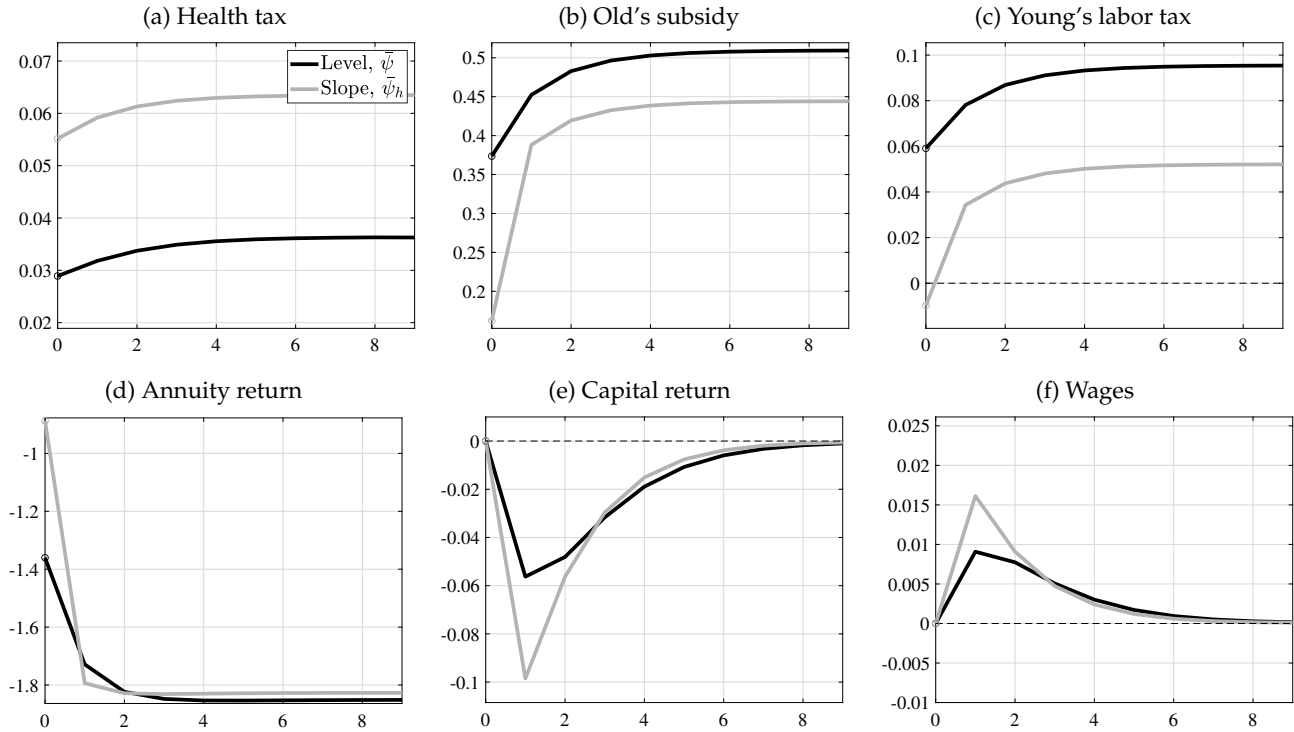
Recall that these social security benefits are funded by income from health taxes and labor taxes from young, $d_t^* \bar{s}_t = \bar{\tau}_t + \tau_t^h h_t^*$. Moreover, the fact that no public debt is needed to implement this optimal policy suggests that there is not sustainability concerns about adjusting the government size to demographic shifts. I discuss more about the adjustment of the policy instrument to demographic shifts below.

Proposition 8 characterizes the government programs necessary to decentralize the planner's allocation via a competitive market equilibrium. To understand how these programs should respond to these demographic shifts, I only show the variation in the optimal government instruments in a world with and without demographic shifts.

Under a rise in survival rate, the optimal allocation features higher lower investment and lower old-age consumption. Because the decline in elderly consumption generally exceeds the decline in medical

expenditures, the demographic externality becomes more severe, requiring a higher Pigouvian tax on health investment to restore efficiency. At the same time, the increase in survival rates lowers the return on annuities (since they now span a longer period), leading to a decline in elderly income. The magnitude of the income loss relative to consumption determines the adjustment in intergenerational transfers. Typically, the sharp decline in annuity returns results in a more pronounced fall in capital income, necessitating larger transfers to the elderly. These additional subsidies must be financed, at least in part, through increased tax revenues from the now more heavily taxed health investments.

Figure 9: Policy Instruments in Response to Increments in Life Expectancy



Note. Black-dashed line is the optimal instrument path without demographic shifts; Black-solid line is the optimal path under Proposition 7; and the gray-solid line is the optimal path under an smooth increment in Ψ .

4.2.2 Capital Subsidy and Pigouvian Correction

Provision of fair annuities and the subsidization of savings are equivalent tools for implementing the efficient allocation.²⁰ Proposition 8 shows that the government can replicate the social planner's allocation by completing financial markets—specifically, by offering actuarially fair annuities that yield a return of $R_{t+1}/\Psi_{t+1}(h_t)$. In doing so, the government effectively insures individuals against longevity risk by pooling them within mortality groups. Alternatively, the same allocation can be achieved through appropriate subsidies to physical capital returns, compensating for the absence of annuity markets. The following lemma formalizes this equivalence.

Lemma 2. *A government as in Assumption 2 can implement the social planner allocation (with Pareto weights $\{\phi_t\}$) by following the intervention in Proposition 8 except for point (iii). Instead, the government does not*

²⁰This equivalence holds independently of other market-based solutions discussed earlier.

complete annuity markets but subsidizes risk-free assets setting the tax rate at $\tau_t^k = 1 - 1/\Psi_{t+1}(h_t^*) < 0$. The intergenerational transfers should adjust to this new regime by satisfying $\tau_t + \tau_t^h h_t^* + \tau_t^k R_t^* k_t^* = d_t^* s_t$.

Throughout this document, I exploit the equivalence between a social security system—implemented through actuarially fair annuities—and subsidies to capital returns to draw insights into the interaction between pension and health systems. However, I assume that the government faces institutional or political constraints that limit its ability to use capital taxation, while retaining greater flexibility to reform or redesign the social security system.

4.3 Second-Best Solutions

The previous solutions, while theoretically insightful, are generally infeasible due to the limitations real-world governments face. In particular, assuming that the government can implement fully state-contingent (idiosyncratic) transfers, observe individual health investments, or operate complete annuity markets is not realistic. In this section, I explore how a more constrained, yet realistic, government might still mitigate market inefficiencies. To simplify the analysis, I isolate each source of inefficiency and examine the design of optimal public interventions that can implement a second-best allocation in response to each specific friction.

4.3.1 Health-Prevention Subsidies to Complete Financial Markets

In this section, I argue that one important rationale for subsidizing health investment (opposing the first-best solution recipe) is to reduce the longevity risk whenever the financial markets do not provide perfect annuities to households and the government cannot solve this problem perfectly. To simplify my analysis, I assume that all households are ex-ante homogenous regarding their labor productivity (i.e., $\bar{z}_t = 1$ for all $t \geq 0$). In this setting, I assume that the government can perform intergenerational transfers (τ_t, s_t) , uniform taxes on health, τ_t^h , and fund any fiscal deficit through public debt $B_{g,t}$. As the government have access to lump-sum taxes and transfers across all individuals, it turns out the public debt is redundant, then throughout this section I assume $B_{g,t} = 0$ for all $t \geq 0$.

Ramsey Problem. The government cannot affect financial markets in any sense, so it only relies on intergenerational transfers to generate an improvement with respect the competitive equilibrium allocation. This government will not achieve the first-best allocation but a second-best one. To me, this is a sensible second-best problem for the current model. Under these constraints, the set of implementable allocations for this government is given by the (i) resource constraint: (20), and (ii) the incentive compatibility constraint:

$$u'(c_{y,t}) = (1 - \delta + f'_{t+1}(k_{t+1})) \Psi_{t+1}(h_t) U'(c_{o,t+1}). \quad (24)$$

Using the primal approach, the second-best problem for a government like the one described above is: Given $k_0 > 0$ and all exogenous transitions, a benevolent government maximizes (19) subject to (20) and (24).

Proposition 9. Let $v_t \geq 0$ be the Lagrange multiplier for (24). A government provided with lump-sum transfers and uniform taxes on health, the optimal policy is characterized by (i) a tax on health given by

$$\tau_t^h = \underbrace{\left(1 + \frac{U''_{t+1}}{U'_{t+1}} R_{t+1} v_t\right) \frac{\chi_{h,t+1}}{R_{t+1}}}_{\text{Externality}} + \underbrace{v_t \left(-\frac{u''_t}{u'_t}\right)}_{\text{Capital subsidy}} - \underbrace{v_t \frac{\Psi'_{t+1}}{\Psi_{t+1}}}_{\text{Completing markets}} \leq 0 \quad (25)$$

and (ii) an optimal intergenerational allocation:

$$\phi_t^{-1}(1 + n_t) = (1 + \omega_{ig,t}) \cdot \frac{u'(c_{y,t})}{U'(c_{o,t})} \text{ where } \omega_{ig,t} \equiv \frac{-\frac{u''_t}{u'_t} v_t - \frac{U''_t}{U'_t} R_t v_{t-1}}{1 + \frac{U''_t}{U'_t} R_t v_{t-1}} > 0. \quad (26)$$

Equation (C.3) reveals that in the above-described economy there are additional motives to distort private health investment decisions than the demographic externality. In particular, taxes on health may also serve as an implicit capital subsidy, while tax subsidies operate as a mechanisms to complete markets whenever there are not perfect annuities. To further study this case let us consider the following special cases.

The usage of intergenerational transfers as a pooling risk tool is also discussed in [Samuelson \(1975b\)](#), [Diamond \(1977\)](#), [Sheshinski and Weiss \(1981\)](#), [Smith \(1982\)](#), [Boadway et al. \(1991\)](#), and [Krueger and Kubler \(2006\)](#).

No Longevity Risk. Suppose that $\Psi_{t+1} = 1$ for all $t \geq 0$. In this economy, annuities are not required so there is not incomplete financial markets. In fact, the equation (24) is not binding ($v_t = 0$) for the second-best problem, meaning that the Ramsey planner is as powerful as the social planner. In fact, in this economy, the competitive equilibrium allocation is efficient.

Exogenous Longevity Risk. Suppose $\Psi_{t+1} \in (0, 1)$ with $\Psi'_{t+1} = 0$. In this economy, the only reason to invest in health is to reduce future medical expenses. However, there is not demographic externality. From Proposition C.2, the optimal tax on health is

$$\tau_t^h = -v_t \frac{u''_t}{u'_t} > 0.$$

Whenever longevity is exogenous but individuals can invest health to reduce future medical health expenditure, taxing health investments is optimal. The main idea is that the IC constraint (24) limits the planner ability to generate a properly steep consumption path within each generation.

4.3.2 Health-Prevention Subsidies as a Redistribution Tool

Suppose now that $\bar{z}_t > 1$ so that inequality is a policy concern. I show that subsidies to health promotes health and longevity equity and, thus, completes the “completing-the-market” rationale to subsidize health investments. Suppose again the government cannot affect financial markets in

any sense, so it only relies on intergenerational transfers to generate an improvement with respect to the competitive equilibrium allocation. This government will not achieve the first-best allocation but a second-best one. In particular, we can show that the second-best Ramsey problem is to choose $\{h_t(z), c_{y,t}(z), c_{o,t}(z), k_{t+1}\}_{t \geq 0}$ to maximize

$$\sum_{t=0}^{\infty} \Phi_t \int [u(c_{y,t}(z)) + \Psi_{t+1}(h_t(z))U(c_{o,t+1}(z))] \partial \Pi_t(z) + \Phi_{-1} \int \Psi_0(h_{-1}(z))U(c_{o,0}(z)) \partial \Pi_{-1}(z)$$

subject to

$$\begin{aligned} u'(c_{y,t}(z)) &= R_{t+1} \Psi_{t+1}(h_t(z)) U'(c_{o,t+1}(z)) \quad \text{for all } z \\ \frac{\Psi'_{t+1}(h_t(z)) U(c_{o,t+1}(z))}{\Psi_{t+1}(h_t(z)) U'(c_{o,t+1}(z))} &= \frac{\Psi'_{t+1}(h_t(z)) U(c_{o,t+1}(z))}{\Psi_{t+1}(h_t(z)) U'(c_{o,t+1}(z))} \\ f_t(k_t) - (1 - \delta)k_t &= \int (c_{y,t}(z) + h_t(z)) \partial \Pi_t + \frac{1}{1 + n_t} \int c_{o,t}(z) \Psi_t(h_{t-1}(z)) \partial \Pi_{t-1} + (1 + n_{t+1})k_{t+1}. \end{aligned}$$

The solution for this Ramsey problem is more convoluted, but I argue that there is an additional rationale for subsidizing health investments that add a new negative term in Equation (C.3). The idea behind this proposition is simple. subsidies to health investment act as a redistributive tool because they directly counteract the unequal ability of individuals to invest in longevity-enhancing health when markets are incomplete and income disparities are large. Since preventive health spending raises survival probabilities and amplifies the long-run returns to saving, richer agents—who face lower liquidity constraints—naturally invest more in health and live longer, accumulating greater wealth over time. Poorer individuals, by contrast, underinvest in health due to tighter budget constraints and limited access to annuities or insurance. A subsidy on health investment effectively transfers resources toward these constrained agents by lowering the relative price of health, increasing their survival prospects, and strengthening their saving incentives. As a result, it simultaneously reduces inequality in both lifespan and wealth, functioning as a targeted redistributive policy that improves welfare without distorting the aggregate efficiency of health–savings decisions.

5 Quantitative Model

Time is discrete and indexed by $t = 0, 1, \dots, \infty$. At each period t , the economy is populated by a continuum of households from different birth cohorts who live for $J + 1$ years as a maximum. The age of each birth cohort is indexed by $j = 0, 1, \dots, J$. For $j \in \mathcal{J}_w = \{0, \dots, J - 1\}$, households participate in the labor market and are called workers, while for $j \in \mathcal{J}_r = \{J, \dots, J\}$, households retire.

A representative firm uses capital, effective labor units, and constant returns to scale production technology to produce final goods. This firm sells output to consumers in a competitive market. A government collects taxes and issues bonds to provide Social Security (SS) benefits for retirees and finance government expenditure. Below, capital letters denote aggregates, while lower-case letters indicate individual variables, per-capita variables or prices.

5.1 Households

Healthcare Technology. The health technology is captured by three aspects of this economy. The first one is the survival probability which depends on the individual's health stock, $\Psi_{j,t}(h, z_p)$. This function captures the economy's health knowledge to avoid death for an individual with a given level of health stock. The second aspect that captures health technology is that the health stock depends on the individuals' actions. In particular, agents can control the evolution of their health stock by investing according to the following cumulative technology

$$h_{j'+1,t'} = (1 - \delta_j^h)h_{j',t'-1} + \mathcal{H}_{j'}(\iota)$$

where $\mathcal{H}_{j'}$ captures the effect of health investments on health stock. Thirdly, health stock affect labor productivity. Finally, the fourth aspect of health technology in the economy is encoded in function $\mathcal{M}_{j',t'}(h)$ which captures the medical costs that an individual with health stock h incurs. It is worth emphasizing that this function only considers out of the pocket curative medical expenses, so that it is affected by the health system regime.

Note that ι is not health investment in prevention, $\iota_{\text{prevent}} = \iota - \iota_{\text{cure}}$ where $\mathcal{H}_{j'}(\iota_{\text{cure}}) = \delta_j^h h_{j',t'-1}$

Preferences. Preferences for all households (workers and retirees) are time-separable with a subjective discount factor $\beta \in (0, 1)$, and the period utility function from the consumption of final goods c is $u(c)$. Agents face idiosyncratic mortality risks. At the end of period t , an agent who is j -years old turns one more year with probability $\Psi_{j,t} \in [0, 1]$ and dies with probability $1 - \Psi_{j,t}$ (with $\Psi_{j,t} = 0$), whereby they derive warm-blow bequests utility, $\mathcal{U}(a)$, where a is the level of wealth the agents own at their death time. Given a path for consumption $c_{j,t} \equiv \{c_{j',t'}\}_{j' \geq j, t' \geq t}$, health stock $h_{j,t} \equiv \{h_{j'+1,t'}\}_{j' \geq j, t' \geq t}$ and assets $a_{j,t} \equiv \{a_{j'+1,t'}\}_{j' \geq j, t' \geq t}$, the expected life time utility for an individual aged j in period t is to maximize

$$U_j(c_{j,t}, h_{j,t}, a_{j,t} | z_s) \equiv \mathbb{E} \left[\sum_{j'=j}^J \beta^{j'-j} \tilde{\Psi}_{j',t'}(h_{j,t}, z_p) \cdot \left\{ u_{j'}(c_{j',t'}) + \omega(h_{j'+1,t'}) \right. \right. \\ \left. \left. + \beta \cdot \left[1 - \Psi_{j',t'}(h_{j'+1,t'}, z_p) \right] \cdot \mathcal{U}(a_{j'+1,t'}) \right\} \middle| z_s \right] \quad (27)$$

where $\tilde{\Psi}_{j',t'}(h_{j,t}, z_p) \equiv \prod_{l=j}^{j'-1} [1 - \Psi_{l,t+l-j}(h_{l+1,t+l-j}, z_p)]$ is the probability to reach age j' at period t' . Note that health stock affects individuals' preferences through two channels: (i) life expectancy and (ii) period utility function.

Productivity. Log-labor productivity, $\ln \epsilon_j(\cdot)$, is the sum of two deterministic components, an age-dependent component ($\ln \epsilon_j(h) = \log \epsilon_j + \zeta_z \cdot h$) which depends on health stock h and cohort-permanent component ($\ln z_p$), and one stochastic component ($\ln z_s$):

$$\ln \epsilon_j(h, z) = \zeta_z \cdot h + \ln \epsilon_j + \ln z_p + \ln z_s$$

where $z \equiv (z_p, z_s)$ and the stochastic component evolves according to

$$\ln z_s = \rho_z \cdot \ln z_{s,-} + \sigma_z \cdot \varepsilon \quad \text{with } \varepsilon \sim \mathcal{N}(0, 1). \quad (28)$$

Beyond the age-dependent and stochastic component, agents differ in their permanent productivity component too: At t , each agent i born with permanent income $z_{p,t}$ and transitory income $z_{s,t}$ which are drawn from (time-varying) distribution $\mathbb{P}_{p,t}$ and (stationary) distribution \mathbb{P}_s , respectively. Because the permanent component comes from the same distribution for each cohort born at t , by the law of large numbers, \mathbb{P}_t^p denotes the t -cohort distribution of permanent income.

Optimization Problem. Given the paths for the after-tax real interest rates, \tilde{r}_t , and wages, w_t , individuals aged j in period t choose $\{c_{j,t}, h_{j,t}, a_{j,t}\}$ to maximize (27) subject to

$$\begin{aligned} c_{j',t'} + a_{j'+1,t'} + (1 + \tau_{j',t'}^h)l_{j',t'} + \mathcal{M}_{j',t'}(h_{j',t'-1}) &= \begin{cases} (1 + \tilde{r}_{t'}) \cdot a_{j',t'} + b_{q,t'} + \mathcal{Y}_{j',t'}(w_{t'}e_{j'}(h_{j',t'}, z)) & j \in \mathcal{J}_w \\ (1 + \tilde{r}_{t'}) \cdot a_{j',t'} + \mathcal{S}_{j',t'}(z_p) & j \in \mathcal{J}_r \end{cases} \\ h_{j'+1,t'} &= (1 - \delta_{j'}^h)h_{j',t'-1} + \mathcal{H}_{j'}(l_{j',t'}) \\ a_{j'+1,t'} &\geq \underline{a}_{j'+1,t'} \end{aligned}$$

for all $t' = t + j' - j$ and $j' = j, j + 1, \dots, J$, $e_{j'}(\cdot)$ denotes labor productivity, $\underline{a}_{j,t}$ are time- and age-varying borrowing limit constraints (with $\underline{a}_{j+1,t} = 0$ for all t), and $\mathbb{I}_{\mathcal{A}}$ is the indicator function for the event \mathcal{A} . Moreover, $\{b_{q,t'}\}_{t' \geq t}$ is the expected path of bequests, and $\{\mathcal{Y}_{j',t'}(\cdot)\}_{j' \geq j, t' \geq t}$ and $\{\mathcal{S}_{j',t'}(\cdot)\}_{j' \geq j, t' \geq t}$ are the path for the after-tax labor income schedule (i.e., net of labor income taxes) and for social security benefits rules, respectively.

Recursive Formulation. To write households' problems recursively, denote $x \equiv (a, h, z)$ as the triple specifying the idiosyncratic states. Furthermore, the vector of aggregate variables is denoted by $X_t = \{\tilde{r}_t, w_t, b_t, \{\mathcal{Y}_{j,t}, \mathcal{S}_{j,t}, \mathcal{M}_{j,t}\}_j\}$. Then, prior to the realization of mortality shock, the continuation value for an individual at idiosyncratic state x is

$$W_{j,t}(x; X) \equiv \begin{cases} \Psi_{j,t}(h, z_p) \mathbb{E}_{z_{s,+}} \left[V_{j+1,t+1}(a, h, z_p, z_{s,+}; X) \Big| z_s \right] + [1 - \Psi_{j,t}(h, z_p)] \mathcal{U}(a) & j = 0, \dots, J_r - 2 \\ \Psi_{j,t}(h, z_p) V_{j+1,t+1}(x; X) + [1 - \Psi_{j,t}(h, z_p)] \mathcal{U}(a) & j = J_r - 1, \dots, J \end{cases}$$

where $V_{j,t}(x; X)$ is the date- t value function for a j -years-old living individual at idiosyncratic state x and aggregate state X which is pinned down by

$$V_{j,t}(x; X_t) = \max_{c, l, a_+ \geq \underline{a}_{j+1,t}} \{u_j(c) + \omega(h_+) + \beta W_{j,t}(a_+, h_+, z; X_{t+1})\} \quad (29)$$

subject to

$$\begin{aligned} c + a_+ + (1 + \tau_{j,t}^h)l + \mathcal{M}_{j,t}(h) &= (1 + \tilde{r}_t) \cdot a + \mathbb{I}_{j \in \mathcal{J}_w} \cdot [b_{q,t} + \mathcal{Y}_{j,t}(w_t e_j(h, z))] + \mathbb{I}_{j \notin \mathcal{J}_w} \cdot \mathcal{S}_{j,t}(z_p) \\ h_+ &= (1 - \delta_{j'}^h)h + \mathcal{H}_{j'}(l). \end{aligned}$$

Since $\Psi_{J,t}(z) = 0$ for any t and z , then the terminal value $V_{J,t}(\cdot)$ can be computed solely based on u and \mathcal{U} .

Demographics and Distribution's Law of Motion. Let $N_{j,t}(x)$ be the number of households aged j at state x and period t . Consequently, $N_t \equiv N_{w,t} + N_{r,t}$ is the population size in the economy at t , where

$$N_{i,t} \equiv \sum_{j \in \mathcal{J}^i} \int_x N_{j,t}(x) \partial x \text{ for } i \in \{w, r\}$$

the total number of workers and retirees, respectively. Each period t , a total of $n_t N_{t-1}$ new workers (0-years-old in the model) born with $a_{t,0}$ assets where n_t is interpreted as the fertility rate at time t . Thus, the economy's population growth rate at time t , $n_t^{\text{pop}} \equiv N_t/N_{t-1} - 1$, is determined by

$$n_t^{\text{pop}} = n_t - \sum_{j=0}^J \int_{\mathbf{x}} [1 - \Psi_{j,t-1}(\mathbf{x})] \mu_{j,t-1}(\mathbf{x}) \partial \mathbf{x} \quad (30)$$

where $\mu_{j,t}(\mathbf{x}) \equiv \frac{N_{j,t}(\mathbf{x})}{N_t}$ is the fraction of people at state \mathbf{x} in period t . The distribution over idiosyncratic states $\mu_{j,t}(\mathbf{x})$ is a crucial object in describing the equilibrium in this economy and is fully described in Appendix D.1.

5.2 The Rest of the Model

Bequests. At the beginning of period t , the resources left by the agents who passed away at the end of period $t-1$ are collected and redistributed uniformly across all agents living at t (including the newborns). Therefore, the level of inheritances received for each agent at t is $B_{q,t}/N_{w,t}$ where

$$B_{q,t} = (1 + \tilde{r}_t) \cdot \sum_{j=0}^J \int_{\mathbf{x}} [1 - \Psi_{j,t-1}(\mathbf{x})] a_{j,t-1}^+(\mathbf{x}) N_{j,t-1}(\mathbf{x}) \partial \mathbf{x} \quad (31)$$

is the aggregate bequests which includes the assets' return.

Final-Good Producers. A continuum of competitive firms produces a homogeneous final good according to neoclassical production technology: $Y = ZF(K, L)$ where Z denotes the total factor productivity, K is the aggregate stock of capital and L is the aggregate effective labor units. Profit maximization leads to a demand for factors of production: $r = ZF_K(K, L) - \delta$ and $w = ZF_L(K, L)$, where $\delta \in (0, 1]$ is the depreciation rate of capital.

Government. At any period t , the government issues bonds (denoted by $B_{g,t+1}$) and collects taxes (on capital income T_t^k , on prevention T_t^h , and on labor income T_t^ℓ) to finance social security benefits S_t , government expenditures G_t , curative medical expenses subsidies M_t , and financial debt costs. Thus, the government budget constraint is

$$S_t + M_t + G_t + (1 + r_t)B_{g,t} = B_{g,t+1} + T_t^k + T_t^h + T_t^\ell. \quad (32)$$

Aggregate revenue from capital and health taxes are

$$T_t^k = \sum_{j \in \mathcal{J}_r} \tau_{j,t}^k r_t \sum_z a(z) \pi_z \quad \text{and} \quad T_t^h = \sum_{j \in \mathcal{J}_r} \tau_{j,t}^h \sum_z h(z) \pi_z,$$

while aggregate revenue from labor income taxes is

$$T_t^\ell = \sum_{j \in \mathcal{J}_w} \int_{z_{p,t-j}} \int_{z_s} \mathcal{T}_{j,t}(w_t e_j(z_{p,t-j}, z_s)) \int_a N_{j,t}(a, z_{p,t-j}^p, z_s^s) \partial a \partial z_{p,t-j} \partial z_s, \quad (33)$$

where labor income tax schedule $\mathcal{T}_{j,t}(\cdot)$, so that $\mathcal{Y}_{j,t}(y^\ell) = y^\ell - \mathcal{T}_{j,t}(y^\ell)$ for labor income y^ℓ , and (iii) public expenditure from medical and pension systems are

$$M = \sum_{j \in \mathcal{J}_r} \tau_{j,t}^h \sum_z h(z) \pi_z \quad \text{and} \quad S_t = \sum_{j \in \mathcal{J}_r} \int_{z_{p,t-j}} \mathcal{S}_{j,t}(z_{p,t-j}) \int_a N_{j,t}(a, z_{p,t-j}) \partial a \partial z_{p,t-j} \quad (34)$$

with SS benefit schedule $\mathcal{S}_{j,t}(\cdot)$.

5.3 Calibration

In this section, I cover the following aspects: the functional forms of the economic variables within the model, the parameterization of these functions, and the computation of exogenous transitions. Our baseline calibration is summarized in Table 1.

Table 1: Baseline calibration

| | DESCRIPTION | VALUE | TARGET/SOURCE |
|---|--|-----------------------|-------------------------------------|
| Demographics and Healthcare | | | |
| n | Fertility rate | 0.03 | Population growth rate = 1% |
| $\bar{\psi}, \bar{\psi}_{\text{age}}, \bar{\psi}_h, \bar{\psi}_p$ | Survival rate parameters | See text | WPP, HRS, MEPS |
| $\bar{M}_0, \bar{\xi}_m, m^{\text{slope}}$ | Medical expense parameters | See text | WPP, HRS, MEPS |
| $\bar{H}_0, \bar{\xi}_h, h^{\text{slope}}$ | Health production parameters | See text | WPP, HRS, MEPS |
| Preferences and borrowing limit | | | |
| σ^{slope} | Elast. of Intertemporal Substitution | Internally calibrated | $\frac{d \log c}{d \log z_p} = 0.7$ |
| o | Scale term in utility function | 0.3 | Straub (2019) |
| Σ | Elast. of Bequest Function | σ_{ret} | Normalization |
| β | Subjective discount rate | Internally calibrated | Interest rate 5.0% (p.a.) |
| \bar{u} | Constant utility from living | 10 | Hall and Jones (2007) |
| \bar{U}_0 | Weight on bequest motives | 10.2 | Bequest/GDP = 5% |
| \bar{U}_1 | Intercept in bequest motives | 1.45 | Inequality measure of bequests |
| a | Borrowing limit | 0.00 | Non-borrowing constraint |
| Production | | | |
| α | Capital share | 0.37 | Straub (2019) |
| δ^k | Depreciation Rate (p.a.) | 5.5% | Straub (2019) |
| (\bar{z}, \underline{z}) | Permanent productivity level | Internally calibrated | Income share of top 10% = 25% |
| $\bar{\xi}_z$ | Labor income sensitivity to health stock | 0.10 | Hosseini et al. (2025) |
| ϵ_j | Age-varying productivity | Internally calibrated | Lagakos et al. (2018a) |
| ρ_z | Income shock persistence | 0.90 | Straub (2019) |
| σ_z^2 | Var. of income shocks | 0.038 | Straub (2019) |
| Government | | | |
| b_y | Debt (% GDP) | 60% | Av. during 1970 – 2020 |
| g_y | Gov. expenditure (% GDP) | 13% | Residual |
| τ^k | Capital income tax rate | 40% | Straub (2019) |
| τ^h | Health investment tax | 0% | Benchmark |
| τ^ℓ | Average income tax | 30% | Straub (2019) |
| λ | Income tax progressivity | 0.18 | Straub (2019) |
| $\mathcal{S}(\bar{y})$ | Social Security tax | $0.4 \cdot \bar{y}$ | Av. replacement rate 40% |

Demographics and Healthcare Technology. I calibrate the fertility rate such that the population growth is 1%. The parametric assumptions that I use for the survival rate is

$$\Psi_j(h, z) = \text{logistic}(\bar{\psi} + \bar{\psi}_{\text{age}}j + \bar{\psi}_h h + \bar{\psi}_p z_p),$$

where $\bar{\psi}_p$ allows me to match longevity disparities as I anticipate that differences in preventive health cannot explain all these disparities observed across income groups. For medical expenses, I assume

that $\mathcal{M}_j(h) = \bar{M}_j \exp(-\xi_m \cdot h)$ where

$$\bar{M}_j = \bar{M}_0 \exp(m^{\text{slope}} j)$$

where m^{slope} controls the growth rate of medical expenses over the life cycle. A similar structure is imposed to health stock production $\mathcal{H}(t) = \bar{H}_j t^{\xi_h}$ where

$$\bar{H}_j = \bar{H}_0 \exp(-h^{\text{slope}} j)$$

where h^{slope} rules the depreciation of health investments' productivity. All of these parameters are not calibrated directly, but parameterized such that the model matches certain patterns observed in the data. Below, I provide further details about this parameterization.

Idiosyncratic Labor Productivity. I use [Straub \(2019\)](#)'s calibration of the stochastic component of the income process (28). However, since my income process is not precisely the same as his, I approximate the process to have the same second moments leading to $\rho_z = 0.90$ and $\sigma_z^2 = 0.038$. Regarding the deterministic component of labor I proceed as follows. First, I assume ϵ_j is quadratic in j and calibrate the parameters of this polynomial to match the life-cycle wage growth documented in [Lagakos et al. \(2018b\)](#). Second, the value for ξ_z is calibrated based on the [Hosseini et al. \(2025\)](#) estimation about the role of health in lifetime income inequality.

Preferences, Borrowing Limit, and Technology. In all our numerical exercises, we use the following functional forms for households' instantaneous utility:

$$u_j(c) = \bar{u} + \frac{(c/o)^{1-\sigma_j} - 1}{1 - \sigma_j} \quad \text{and} \quad U(a) = \bar{U}_0 \frac{((a + \bar{U}_1)/o)^{1-\Sigma} - 1}{1 - \Sigma}$$

where σ_j controls for the Elasticity of Intertemporal Substitution (EIS) and \bar{U}_0 and \bar{U}_1 are parameters governing the value of bequests. In line with [Hall and Jones \(2007\)](#), the parameter \bar{u} is calibrated to match the estimations of the value of statistical life.

Moreover, I follow [Straub \(2019\)](#) for the calibration of o , σ_j and Σ . In particular, $\sigma_{j+1} = \sigma^{\text{slope}} \cdot \sigma_j$ for workers, $\sigma_{j+1} = \sigma_j$ for retirees, and $\Sigma = \sigma_{\text{ret}}$. As explained below, σ^{slope} is calibrated to match the concavity of the consumption function in income which was estimated by [Straub \(2019\)](#). The strategy for the parameterization of \bar{U}_0 and \bar{U}_1 are also provided below.

The aggregate production function is $F(K, ZL) = K^\alpha (ZL)^{1-\alpha}$, where α is the labor share of output. In line with national accounts, the capital share of output is set at 0.37 and the p.a. capital depreciation rate at 5.5%, which is borrowed from [Straub \(2019\)](#).

Government and Social Security. The debt-GDP ratio are set to their historical means during 1950 – 2017, which, according to FRED, is 60%, respectively. The social security tax and benefit, τ and s , are calibrated such that the average replacement rate equals 40% as it is common in social security studies (e.g., see [De Nardi \(2004\)](#)). Some other aspects for the government policy instruments are taken from [Straub \(2019\)](#).

Estimation. The estimation of the model is as follows. Given a certain value for

$$\Theta_{\text{estimation}} \equiv \left(\zeta_m, m^{\text{slope}}, \bar{M}_0; \zeta_h, h^{\text{slope}}, \bar{H}_0; \bar{\psi}, \bar{\psi}_{\text{age}}, \bar{\psi}_h, \bar{\psi}_p \right)^\top,$$

I calibrate $\Theta_{\text{match}} \equiv (\beta, \sigma^{\text{slope}}, Z, \bar{U}_0, \bar{U}_1)$ to match the following targets: (i) interest rate, (ii) parameter ϕ in Straub’s regressions, (iii) bequest flow over GDP of 5%, (iv) a share of bequest at age $J = 65$ by the top 10% of 90%, and (v) normalization of the aggregate GDP to one.

Next the parameter vector $\Theta_{\text{estimation}}$ is estimated by a method of simulated moments that targets (i) preventive health measure profiles (as described in Section 2), (ii) total medical expenses as described in Section 2, (iii) survival rate by income and health, and (iv) income gap by health.

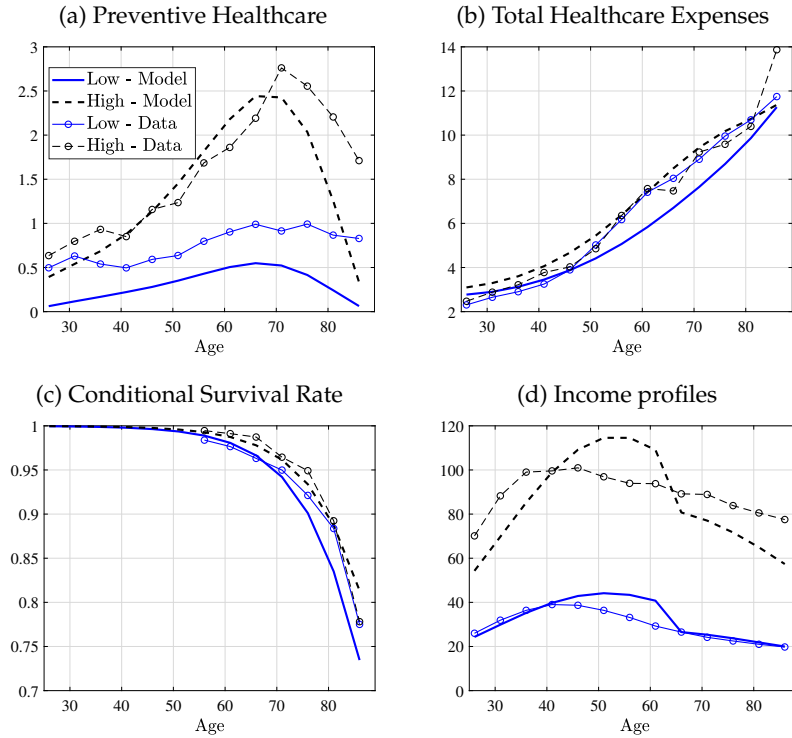
5.4 Calibration Results

Data fit. Figure 10 compares key life-cycle patterns generated by the calibrated model with their empirical counterparts for individuals in the bottom and top parts of the income distribution. Panel (a) shows that the model captures the broad income gradient in preventive health investment: higher-income agents invest substantially more in preventive healthcare over the life cycle, and the profile increases steadily with age before tapering off late in life. While the model somewhat exaggerates the steepness of this rise for high-income individuals, it successfully reproduces both the qualitative shape and the widening gap between income groups observed in the data. Panel (b) presents total medical expenditures and shows that the model generates a strong upward trajectory with age, consistent with empirical patterns. Although the model overpredicts late-life spending among high-income households, it matches the timing and ordering of the profiles reasonably well, with low-income individuals exhibiting lower expenditures throughout.

Panels (c) and (d) compare survival probabilities and labor-income profiles across income groups. Panel (c) demonstrates that the model replicates the well-documented longevity gradient: higher-income households experience better survival probabilities at every age, and survival declines accelerate sharply after the mid-60s, as in the data. The model tracks the empirical patterns closely, especially for low-income individuals, although it slightly underestimates late-life survival for the high-income group. Finally, panel (d) shows that the model reproduces the broad features of income dynamics, including the hump-shaped profile and the persistent gap between income groups. While the calibration cannot match the full magnitude of the empirical dispersion, it aligns well with the timing of peak earnings and the relative slopes before and after mid-career. Overall, the model generates life-cycle patterns that are broadly consistent with the data and captures the key heterogeneity necessary for analyzing the endogenous longevity channel.

Stationary Distribution. Figure 11 shows the stationary distributions of assets and health across workers, retirees, and income groups. Retirees hold more assets than workers, reflecting life-cycle saving motives, while high-permanent-income households display a much more right-skewed wealth distribution with a thicker upper tail. Low-income households cluster near low asset levels, consistent with tighter financial constraints and weaker incentives to save. Overall, the model generates realistic

Figure 10: Model-data fit



patterns of wealth inequality across both age and income groups.

The health-stock distributions display similarly meaningful heterogeneity. Workers have higher health stocks than retirees, capturing the natural decline in health with age, and high-income households maintain substantially better health than low-income households. This gap reflects stronger incentives and greater ability to invest in preventive health. Together, the distributions illustrate that the model captures the joint heterogeneity in wealth and health that is central to the endogenous longevity mechanism.

6 Quantitative Results

In this section, I quantitatively assess the implications of endogenous longevity discussed in Section 3 and Section 4. I conduct two main exercises. First, on the positive side, I examine how general equilibrium outcomes respond to exogenous shifts in demographics (specifically changes in survival rates and fertility) and in the income distribution. I pay particular attention to how much these forces contribute to movements in the real interest rate. Second, I evaluate the normative implications by quantifying the welfare effects of alternative health tax policies.

Figure 11: Asset-Health Stock Distribution



6.1 General Equilibrium Effects and Contributions to r^*

Starting from the benchmark calibration, where the stationary equilibrium real interest rate equals 4%, I examine how the interest rate adjusts in a new stationary equilibrium after exogenous changes in demographics and in the dispersion of labor productivity. The quantitative exercise considers three types of exogenous shifts:

1. *Rise in permanent income inequality.* I simulate an increase in the share of total income accruing to the top 10% of the distribution. Specifically, I raise this share from 25% to 35%, consistent with the historical evolution documented in [Piketty and Saez \(2003\)](#) and [Straub \(2019\)](#).
2. *Rise in life expectancy.* According to the World Population Prospects, adult life expectancy in advanced economies has increased by roughly 10 years over the past five decades. To capture this trend, I increase the parameter $\bar{\psi}$ in the survival function, which directly shifts the survival profile upward and generates a comparable rise in life expectancy.
3. *Decline in fertility rate.* I reduce the fertility rate so that the resulting stationary equilibrium features a population growth rate of 0.2%, a value in line with the recent U.S. experience reported in the World Population Prospects.

Table 2 reports the equilibrium responses of the real interest rate to each exogenous movement under

three versions of the quantitative model. The first version, “Exogenous Longevity,” assumes that the health stock does not affect consumers’ utility or budget constraints, eliminating any motive for preventive health investment. Permanent income inequality, however, remains non-neutral because preferences over consumption and saving are non-homothetic due to age-varying σ_j (as in [Straub \(2019\)](#)) and non-homothetic bequest motives (as in [De Nardi \(2004\)](#)). For each scenario, the first column in Table 2 reports the level of the real interest rate in the new stationary equilibrium, and the second column reports the change relative to the benchmark equilibrium.

As emphasized by [Straub \(2019\)](#), an exogenous rise in income inequality increases aggregate asset demand and therefore lowers the equilibrium interest rate. In my model, this mechanism generates a decline of roughly 85 basis points. In line with other strands of the literature (see [Carvalho et al. \(2016\)](#), [Auclert et al. \(2025\)](#)), the exogenous increase in life expectancy and the decline in fertility also contribute meaningfully to the downward trend in interest rates, reducing r by approximately 53 and 102 basis points, respectively.

Table 2: Contributions to the Real Interest Rate Movements in Equilibrium

| | Exogenous Longevity | | Endogenous Longevity | | Full Health | |
|-------------------|---------------------|--------------|----------------------|--------------|-------------|--------------|
| | Level (%) | Diff. | Level (%) | Diff. | Level (%) | Diff. |
| Initial Eq. | 4.00 | - | 4.00 | - | 4.00 | - |
| Inequality | 3.15 | -0.85 | 3.23 | -0.77 | 3.39 | -0.61 |
| Survival rate | 3.47 | -0.53 | 2.75 | -1.25 | 3.97 | -0.03 |
| Fertility | 2.98 | -1.02 | 3.09 | -0.91 | 3.15 | -0.85 |
| All shocks | 1.87 | -2.13 | 1.30 | -2.70 | 2.15 | -1.85 |

The second version of the model incorporates endogenous longevity by allowing health investments to affect individuals’ survival probabilities. This is the natural quantitative counterpart of the framework developed in Section 3. As discussed there, the endogenous longevity channel introduces additional concavity into the asset policy function (at least for low- and middle-income households) which reduces their marginal propensity to save out of permanent income. Consequently, the equilibrium effect of rising income inequality is attenuated once endogenous longevity is introduced: the decline in the interest rate falls from 85 basis points to 77 basis points.

Consistent with the positive analysis, exogenous improvements in survival rates have a stronger impact on aggregate saving in this version of the model. Higher longevity reduces the need for private preventive health investment, freeing resources for financial saving and thereby generating a crowding-in effect. As a consequence, the decline in interest rate due to survival rate is 125 basis points (higher than the 53 basis point contribution in the “Exogenous Longevity”). Finally, the interest-rate response to lower fertility becomes more muted, with a reduced contribution of 91 basis points (relative to 102 basis points in the “Exogenous Longevity”). When longevity is endogenous, health and financial saving become complementary, which increases the sensitivity of households’ asset positions to interest-rate movements and dampens the equilibrium decline in r .

To assess the robustness of the quantitative implications of the endogenous longevity channel, I also simulate the same exogenous changes in a richer model where the health stock affects

additional dimensions of individual behavior (including medical expenditures, labor productivity, and utility) beyond its effect on survival. As discussed in Section 3, these additional channels further increase the concavity of the asset policy function, which strengthens the dampening effect of endogenous longevity on the savings response of low- and middle-income households. As a result, the contribution of rising income inequality to the decline in the interest rate falls from 77 basis points in the “Endogenous Longevity” model to 61 basis points in this “Full Health” model. Because complementarities between health and financial savings become even stronger, asset demand is more sensitive to the interest rate, which in turn reduces the contribution of the fertility decline to the fall in r .

Under this “Full Health” model, however, the response to an exogenous increase in survival is markedly different: its contribution to the decline in the interest rate drops from 125 basis points in the “Endogenous Longevity” model to only 3 basis points. This drastic attenuation is driven by the presence of unavoidable curative medical expenditures. A higher $\bar{\psi}$ raises expected longevity and therefore increases the aggregate burden of curative medical spending. Regardless of how these expenses are financed, the resulting “income effect” reduces the resources available for other utility-generating consumption. In addition, the decline in preventive health investment triggered by the exogenous improvement in survival further increases reliance on curative care, amplifying this expenditure pressure. Together, these forces essentially eliminate the positive savings response associated with longer life expectancy in the simpler model.

6.2 Welfare Consequences of Preventive Health Taxes

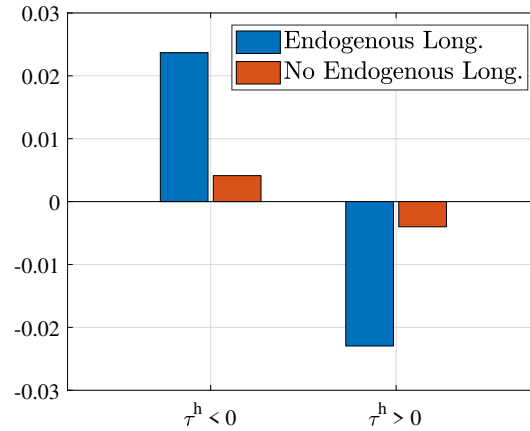
The ability of individuals to influence their own longevity raises a natural policy question: should health investment be taxed or subsidized? In this subsection, I quantify the welfare effects of alternative health tax policies using the calibrated model. Health taxes modify the private incentives to invest in longevity and interact with the distortions arising from incomplete annuity markets, income heterogeneity, and differential health responses across the distribution. The analysis evaluates whether corrective taxation can improve efficiency and examines the distributional implications of such reforms. In particular, I study the welfare consequences of (i) a rise in τ^h of 250 basis points and (ii) a decline in τ^h of 250 basis points.

I use the equivalent consumption measure for evaluating the welfare consequences of this policy reform. In particular, for an aggregate measure of the welfare consequences, I compute Δ solving

$$\sum_{j,z} \int_{a,h} \mathbb{E}_z[U_j((1 + \Delta)c, \mathbf{h}, \mathbf{a}) | a, h, z] \cdot \mu(a, h, z) \cdot \partial a \partial h = \sum_{j,z} \int_{a,h} V_j(a, h, z) \cdot \mu(a, h, z) \cdot \partial a \partial h.$$

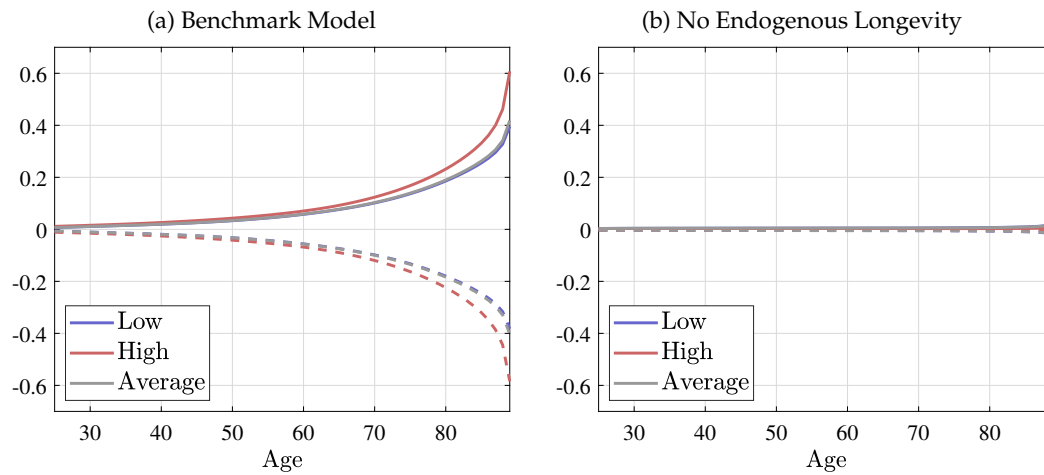
Thus, Δ measures the percentage of consumption the agents are willing to pay for not facing the policy the reform. As this measure does not depend on any characteristics of individuals, this can be interpreted as an aggregate measure. Figure 13 shows this aggregate measures for the two different policy reforms I consider. The blue bars deploy the aggregate welfare consequences for a model with endogenous longevity, while the orange bars show the measures when the endogenous longevity channel is mute.

Figure 12: Aggregate Measures



Similar measures can be computed for different states. In Appendix D.3, I discuss the exact definitions of these measures, but in this section I discuss the measures for Δ_j and $\Delta_j(z_p)$. Figure 13 plots lifetime welfare as a function of age in the baseline economy. Welfare is strictly declining with age, reflecting both discounting and the progressive erosion of health and survival prospects over the life cycle. Differences across income groups are largest at younger ages and gradually vanish as agents approach the end of the lifespan, when the remaining horizon is short and there is little scope for further adjustment in health or saving. This profile provides a useful benchmark for the normative analysis: changes in health taxes will shift the entire age-welfare schedule, and the magnitude and shape of those shifts across ages and groups summarize who gains and who loses from a given reform.

Figure 13: Welfare Consequences



Note. Solid lines are the results for the experiment of a rise of τ^h in 250 basis points, while the dashed lines correspond to a fall of τ^h in 250 basis points.

7 Conclusions

This paper develops a unified framework linking health investments, longevity, and macroeconomic outcomes. By allowing individuals to endogenously choose their survival probability through preventive health spending, the model reveals how health and financial decisions are jointly determined and how their interaction generates non-linear patterns of saving and inequality across income levels. At low incomes, health behaves as a luxury good, crowding out savings, whereas at higher incomes, longer expected lifespans strengthen the saving motive and increase aggregate wealth accumulation. This endogenous longevity channel thus provides a micro-founded explanation for heterogeneity in saving behavior and its aggregate implications.

Building on this mechanism, the paper explores the normative dimension of health and longevity. In the first-best allocation, efficiency requires complete markets for annuities and medical insurance, as well as a Pigouvian tax on preventive health investments to internalize the fiscal externalities of longer lives. When such markets are incomplete, the second-best allocation reverses this logic: health subsidies become welfare-enhancing by mitigating the under-investment in health that stems from uninsured longevity risk and income inequality. Hence, optimal policy simultaneously redistributes resources and restores efficiency, aligning private and social incentives in the presence of endogenous mortality.

Quantitatively, the framework can rationalize key macroeconomic trends, such as the effects of inequality and demographic change on aggregate saving and real rates, while providing guidance for the design of integrated health and pension systems. The results emphasize that longevity is not merely a demographic outcome but an economic choice with aggregate and policy consequences. Recognizing the dual nature of health, as both a consumption good and a survival investment, is essential for understanding how modern economies age, accumulate wealth, and distribute welfare across generations.

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Appendix

A Data and Additional Analysis

A.1 Data Sources

Medical Expenditure Panel Survey. The Medical Expenditure Panel Survey (MEPS) is a nationally representative longitudinal survey of U.S. households conducted by the Agency for Healthcare Research and Quality. It provides detailed information on healthcare expenditures, utilization of medical services, health status, and insurance coverage. MEPS records all payments for healthcare services—including out-of-pocket spending and reimbursements from private insurance, Medicaid, Medicare, and other public or private sources—at the individual level. Each household is interviewed five times over a two-year period, allowing consistent reconstruction of annual medical spending and health utilization patterns.²¹ In this study, MEPS is used to construct average total and service-specific medical expenditures across different age–income groups.

Health and Retirement Survey. The Health and Retirement Study (HRS) is a nationally representative longitudinal survey of U.S. adults over age 50, conducted by the University of Michigan’s Institute for Social Research and funded by the National Institute on Aging. Fielded biennially since 1992, it provides a comprehensive panel dataset with detailed information on health, mortality, income, wealth, demographics, and life-cycle behavior. A key strength of the HRS is its verification of death events through multiple sources—including exit interviews with relatives and linkages to the National Death Index—making it reliable for mortality analysis. This study uses the RAND Harmonized HRS (1992–2020), which offers consistent variable definitions across waves, to construct average measures of conditional survival, wealth, and health across different age–income groups.

American Time Use Survey. The American Time Use Survey (ATUS) is a nationally representative survey conducted by the U.S. Bureau of Labor Statistics that measures how individuals allocate their time across daily activities. Since 2003, it has collected detailed 24-hour activity diaries for a rotating sample drawn from the Current Population Survey (CPS), providing rich information on the time dimension of economic and health-related behaviors. Although ATUS does not track monetary expenditures, it complements datasets like MEPS by documenting nonmarket health investments—such as time spent on exercise, meal preparation, sleep, and caregiving—as well as leisure, work, and household production. The survey’s detailed activity codes, combined with demographic and labor-force characteristics, enable analysis of how time-use patterns vary by income, age, gender, and employment status, offering a unique perspective on the behavioral and opportunity-cost dimensions of health production and well-being.

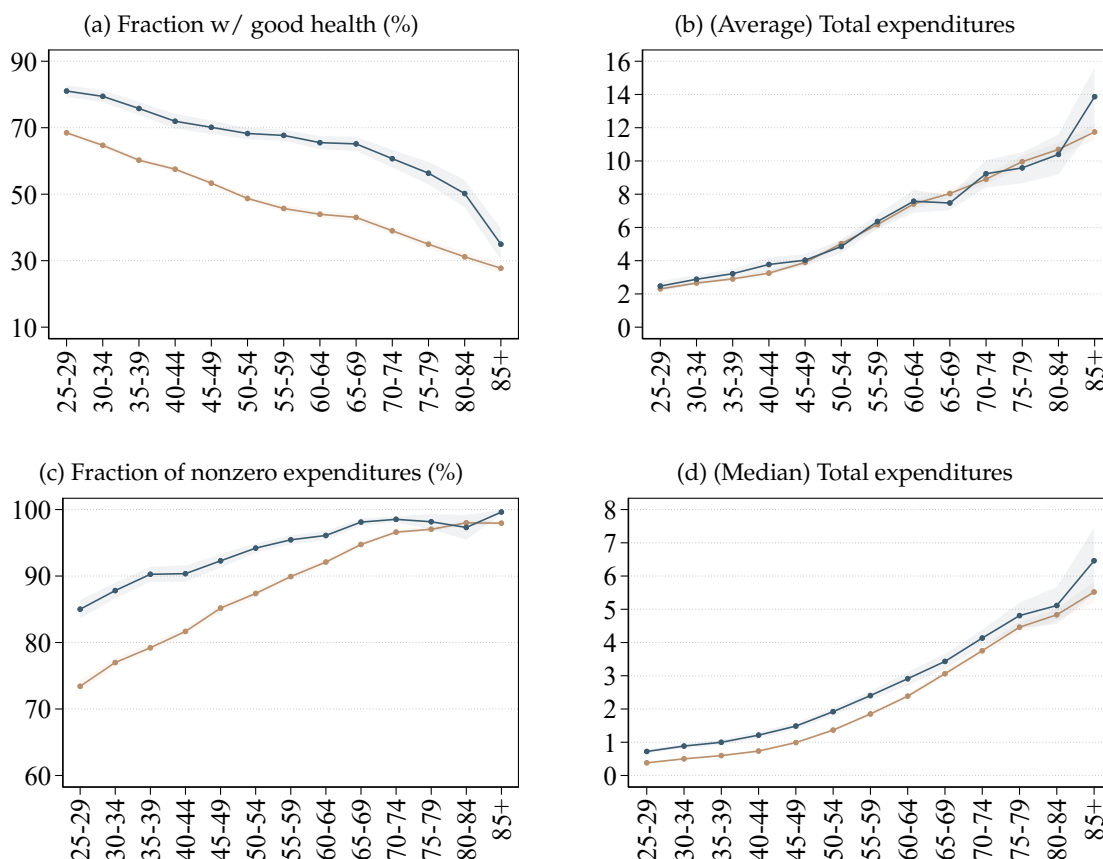
²¹The dataset excludes expenditures on over-the-counter medications.

A.2 Healthcare Expenditures

Figure A.1 presents evidence from the MEPS on the relationship between health status and healthcare spending across income groups throughout the adult life cycle. The figure extends the analysis beyond older populations and shows that income-related health disparities emerge early in adulthood and persist through old age. Each panel compares outcomes for the top 10 percent and bottom 90 percent of the income distribution, highlighting how socioeconomic differences shape both health status and patterns of medical spending.

Panel (a) displays the fraction of individuals reporting good or very good health by five-year age groups. The share in good health declines steadily with age for both groups, yet remains consistently higher among higher-income individuals. The gap is already visible in the late twenties and widens with age, illustrating that health-income gradients are a persistent feature across the life course, not only among older adults as seen in HRS data. These disparities likely reflect cumulative differences in access to preventive care, work conditions, and lifestyle factors that compound over time.

Figure A.1: Health and Healthcare Expenditures



Note. Expenditure variables are in thousands USD. Income groups: Top 10% percent and Bottom 90% percent.

Panel (b) plots average total medical expenditures (in thousands of USD) by age. Average spending rises with age for both groups, especially after age 50, but the differences between income groups

are relatively modest. This apparent similarity in means, however, masks important heterogeneity in spending patterns: many individuals report zero or negligible expenditures within a given year, while a smaller share of the population accounts for very high costs. Panel (c) quantifies this heterogeneity by showing the fraction of individuals with nonzero medical expenditures. The share of positive spenders increases with age for both groups but remains systematically higher among higher-income individuals, suggesting that wealthier households are more likely to engage in healthcare utilization—possibly reflecting better insurance coverage, greater use of preventive and elective services, and fewer financial barriers to care. Panel (d) reports the median of total annual medical expenditures, which provides a more representative measure for the skewed distribution of healthcare spending. Median spending increases sharply after midlife and reveals a clearer income gradient than the mean: the top income group consistently reports higher median outlays, and the gap widens at older ages. This pattern implies that, even if catastrophic expenditures are concentrated among a few high-cost individuals, systematic differences in typical (median) medical spending persist across the income distribution.

In Appendix A, I show that income disparities are especially pronounced for categories of spending associated with preventive and routine healthcare, such as outpatient visits and dental care, while spending on acute or emergency services exhibits smaller differences. Taken together, the evidence from MEPS underscores that health and healthcare inequality extend well beyond old age and that differences in both access and intensity of health investments likely contribute to the persistent health and longevity gradients observed later in life.

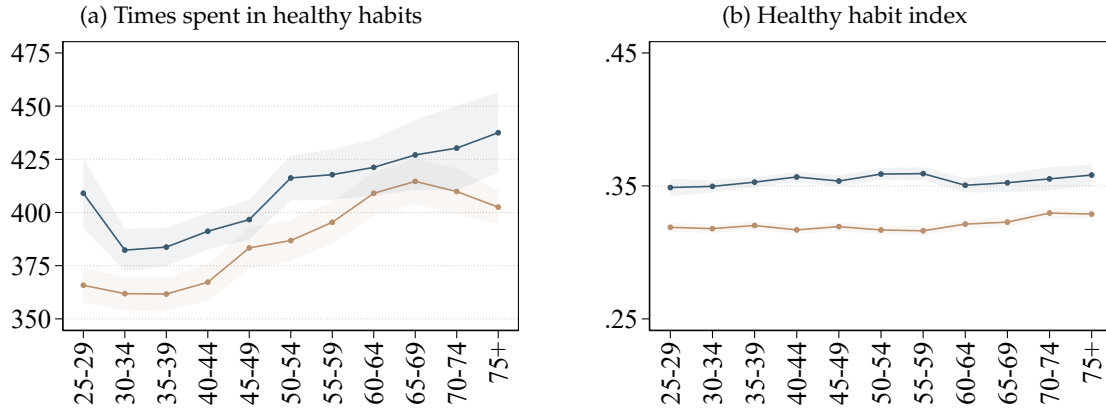
A.3 Disparities in Healthy Activities from ATUS

Figure A.2 presents evidence on non-pecuniary health investments over the life cycle using data from the ATUS. While figures in the main text focused on pecuniary (monetary) healthcare expenditures, this figure highlights a complementary dimension of health investment—the time individuals allocate to health-promoting behaviors. These activities include exercise, personal care, and routine medical visits, all of which represent preventive inputs that improve or sustain health independently of direct financial spending.

Panel (a) reports the total annual hours spent in health-related activities by age and income group. Time devoted to these behaviors increases gradually with age, rising from roughly 370–400 hours per year among individuals in their late twenties to over 400–450 hours among those above age 70. Across the life cycle, individuals in the top 10 percent of the income distribution consistently allocate more time to health-promoting activities than those in the bottom 90 percent, with differences ranging from 20 to 40 hours per year. The increase at older ages partly reflects greater health awareness and more frequent engagement with preventive or maintenance behaviors, but the persistent gap suggests that higher-income individuals sustain healthier time-use patterns even when accounting for differences in work hours or leisure constraints.

Panel (b) summarizes this dimension through a healthy-habit index, defined as the average of three

Figure A.2: Non-Pecuniary Healthcare Expenditure over the Life Cycle



Note. Time is in hours per year. Income groups: Top 10% percent and Bottom 90% percent.

binary indicators that equal one if the individual spends any positive time on exercise, self-care, or medical visits on a given day. That is,

$$\text{Health-Habit} = \frac{\mathbb{1}(t_{\text{sports}} > 0) + \mathbb{1}(t_{\text{selfcare}} > 0) + \mathbb{1}(t_{\text{medvisits}} > 0)}{3}.$$

This measure captures the extensive margin of preventive time use—the likelihood of engaging in any health-promoting behavior rather than the total intensity of effort. The index remains relatively stable across age groups but exhibits a consistent income gradient: higher-income individuals are 5–10 percentage points more likely to report engaging in at least one healthy activity on an average day.

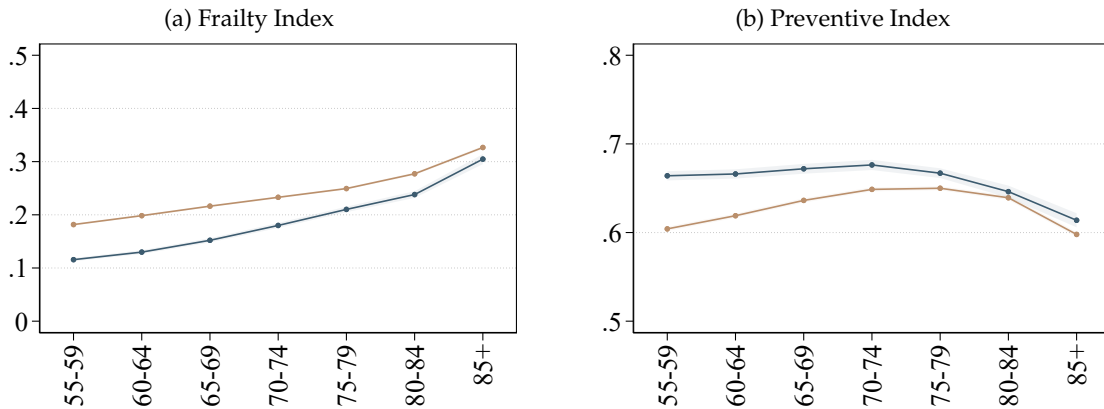
Taken together, the two panels indicate that income-related disparities in non-pecuniary health investments emerge early in adulthood and persist throughout the life cycle. While financial resources influence access to medical care and insurance coverage, these patterns show that behavioral and time-allocation choices also play a central role in shaping health inequality. The findings complement those based on MEPS data by demonstrating that the inequality in preventive inputs extends beyond monetary dimensions, reinforcing the interpretation of health as a form of capital jointly produced through both pecuniary and non-pecuniary investments.

A.4 Other Health Measures from HRS

Frailty and Preventive Index. Figure A.3 reports age profiles of a frailty index and a preventive index constructed using the HRS, separately for individuals in the top 10 percent and the bottom 90 percent of the income distribution. Panel (a) shows that frailty increases steadily with age for both groups, but is systematically lower among higher-income individuals at all ages, with the gap widening in later life. Panel (b) displays the preventive index, which is higher for the top 10 percent throughout most of the life cycle and follows a hump-shaped profile, peaking around ages 65–74 before declining at older ages. Together, these patterns indicate that higher-income individuals engage more intensively in preventive behaviors earlier in life and enter old age in better health, consistent with the presence

of persistent socioeconomic gradients in health investment and health outcomes.

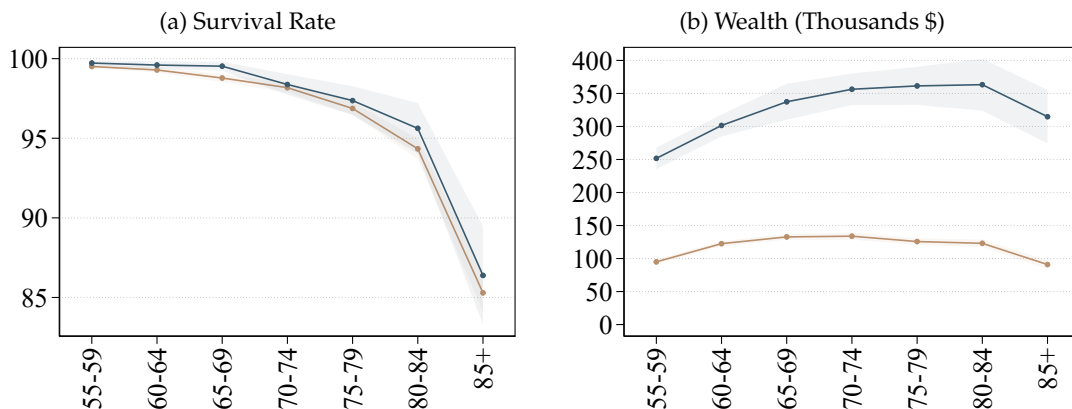
Figure A.3: Frailty and Preventive Index



Note. Income groups: Top 10% percent and Bottom 90% percent.

Longevity and Wealth Gradients in Income. Figure A.4 documents life-cycle gradients in survival and wealth by income group. Panel (a) shows survival rates by age for individuals in the top 10 percent and bottom 90 percent of the income distribution. Survival declines with age for both groups, but remains consistently higher among higher-income individuals, with the gap becoming more pronounced at older ages. Panel (b) displays average wealth holdings over the life cycle. Higher-income individuals accumulate substantially more wealth at all ages, with wealth peaking in late working life and declining thereafter, while the bottom 90 percent exhibit much lower and flatter wealth profiles. Taken together, these patterns highlight a strong joint gradient in wealth and longevity across income groups, consistent with the idea that higher resources are associated with both greater saving and higher survival probabilities.

Figure A.4: Longevity and Wealth



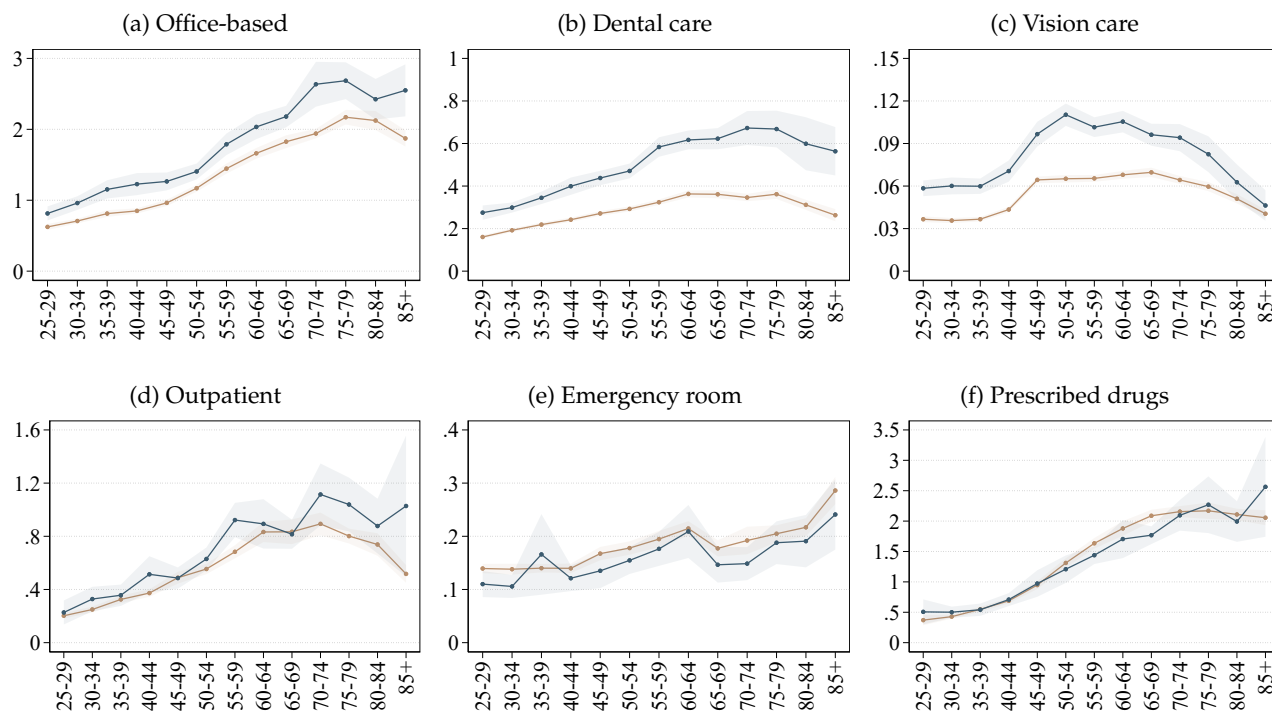
Note. Income groups: Top 10% percent and Bottom 90% percent.

A.5 Healthcare Expenditure by Medical Service

The differences in healthcare expenditures across income groups become more pronounced when disaggregated by type of medical service. As shown in Figure A.5, individuals in the top income decile spend significantly more than those in the bottom quintile on categories that are often associated with preventive health services (e.g., office-based visits, dental care, and vision care). These gaps widen particularly after midlife, suggesting that higher-income individuals invest more consistently in preventive care throughout the life cycle.

In contrast, expenditures on emergency room visits, typically a form of reactive care, are relatively similar across income groups and even slightly higher for the bottom quintile at older ages. Prescribed drug expenditures also show a different pattern: while they rise for both groups with age, lower-income individuals tend to spend more than their higher-income counterparts in later life. Overall, the figure suggests that preventive-oriented services (like office-based, dental, and vision care) are likely the main drivers of the income-based expenditure disparities, highlighting the unequal access to or utilization of proactive health investments over the life cycle.

Figure A.5: Healthcare Expenditure over the Life Cycle: By Medical Services

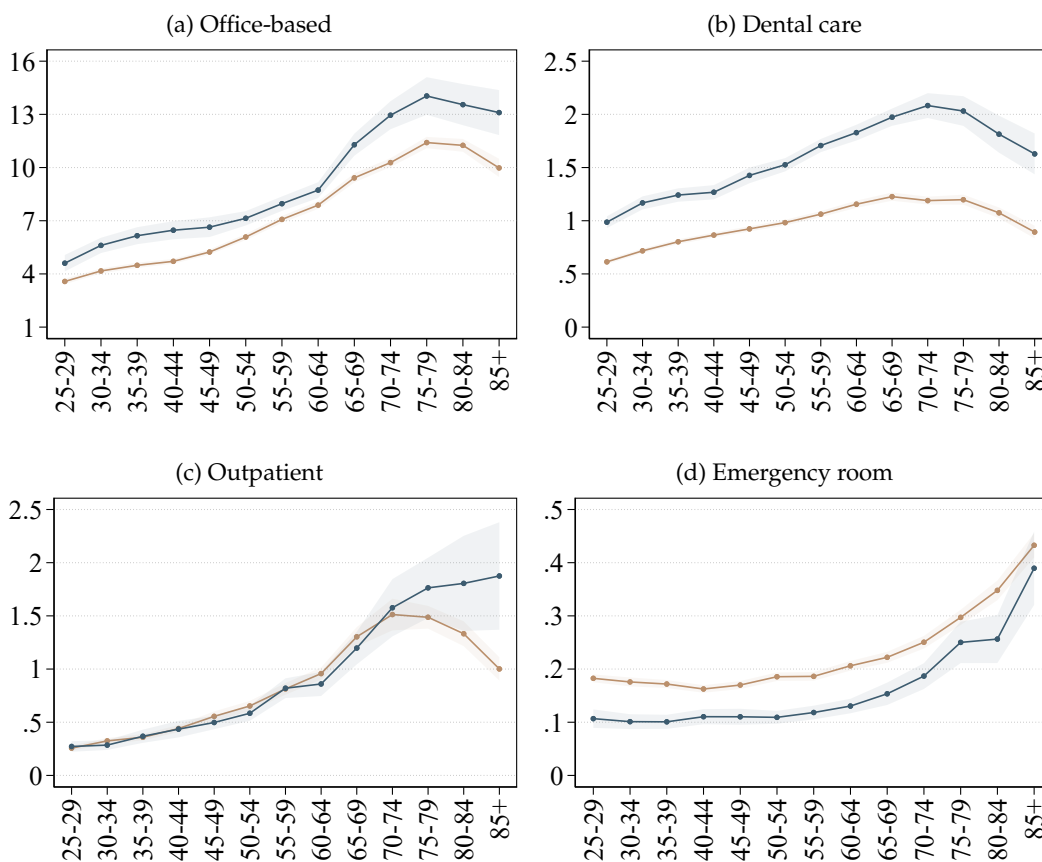


Note. Income groups: Top 10% percent and Bottom 90% percent.

Figure A.6 reports life-cycle profiles of healthcare utilization by income group and type of medical service. The top panels show office-based visits and dental care, which are primarily preventive in nature. In both cases, individuals in the top 10 percent of the income distribution consistently use these services more intensively at all ages, with gaps that widen in mid and late adulthood. By contrast, the bottom panels display outpatient visits and emergency-room use, which are more closely related to

curative or acute care. Here, utilization levels are similar across income groups, and in the case of emergency-room visits, lower-income individuals use these services at least as much, particularly at older ages. Taken together, these patterns indicate that income gradients in healthcare usage are driven mainly by preventive investments rather than by differences in curative care, supporting the view that higher-income individuals devote more resources to health investments that improve long-run survival rather than to the treatment of acute

Figure A.6: Healthcare Usage/Visits over the Life Cycle: By Medical Services



Note. Income groups: Top 10% percent and Bottom 90% percent.

A.6 Age Effects

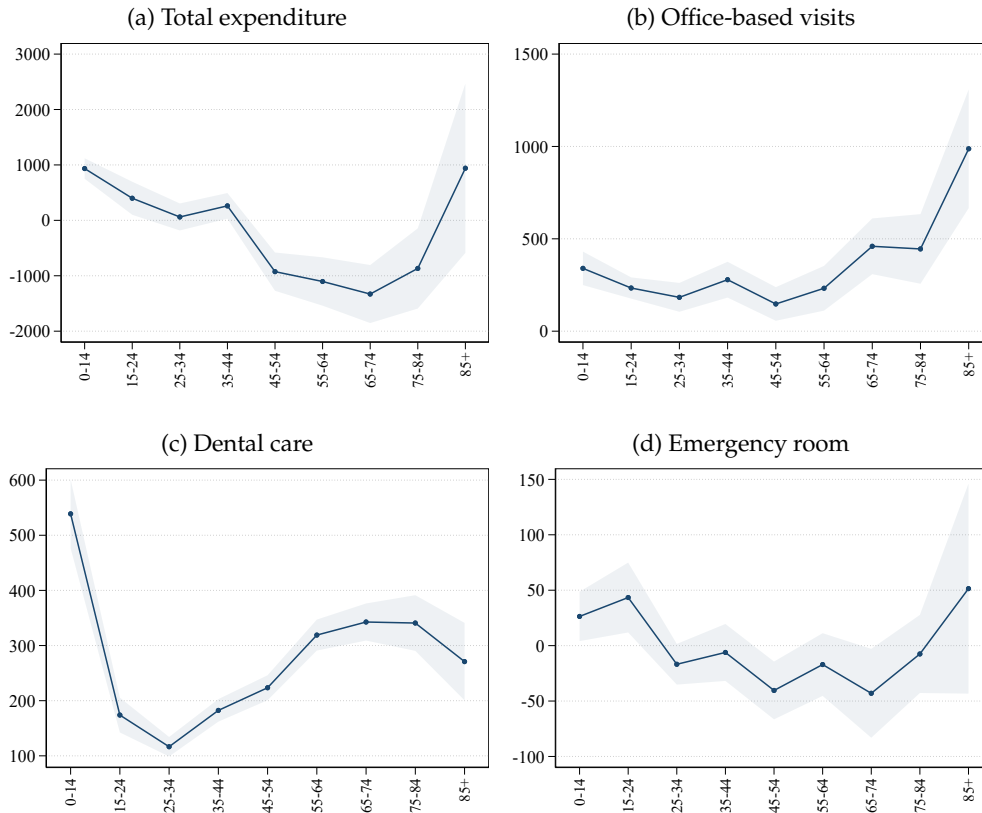
To describe the evolution of healthcare expenditures over the life cycle, I estimate the following regression specification:

$$y_{it} = \sum_j \sum_{k \neq 1} \beta_{jk} D_{ij}^{\text{age}} D_{ik}^{\text{inc}} + \gamma_t + \gamma_{\text{cohort}} + \delta^\top X_{it} + \varepsilon_{it} \quad (\text{A.1})$$

y_i is a measure of health expenditure (e.g., total spending or category-specific spending) for individual i at time t , and X_{it} includes demographic and socioeconomic controls. The indicators D_{ik}^{age} and D_{ik}^{inc} denote membership in age group j and income group k , respectively, with the first income

quintile omitted as the reference group. Then, the interaction terms $D_{ij}^{\text{age}} D_{ik}^{\text{inc}}$ allow for a flexible characterization of life-cycle expenditure profiles across income groups. The specification also includes year fixed effects γ_t , cohort fixed effects γ_{cohort} , and an error term ε_{it} . This framework allows me to extract the age-specific expenditure patterns by income group, isolating how preventive and total healthcare investments vary over the life course while controlling for cohort and time effects.

Figure A.7: Healthcare expenditure over the life cycle: The age affect



The results are given by the estimated coefficients $\hat{\beta}_{jk}$, which capture the differential effect of being in age group j and income group k (relative to the baseline income group). These coefficients are shown in Figure A.7 and trace out the age profiles of healthcare expenditure across income groups, controlling for time trends, cohort effects, and individual-level characteristics. The figure plots a similar pattern as the one revealed in Figure A.5: Preventive-oriented services (like office-based, dental, and vision care) are likely the main drivers of the income-based expenditure disparities.

B Omitted Derivations for Section 3

B.1 Satiation Indices

Satiation indices play an important role in the characterization of the individual behavior. I heavily use the following lemma that derives relationship between several elasticity measures for a differentiable function F . Recall that in the main text, I define the semi-elasticity of F as $\vartheta_F(x) \equiv \frac{F'(x)}{F(x)}$ and the elasticity of any function f as $\eta_f(x) \equiv \frac{f'(x)x}{f(x)}$.

Lemma B.1. *Consider a function $F \in C^3(\mathbb{R})$ with $F' > 0$, $F'' < 0$ and $F''' \geq 0$. The elasticity of the normalized marginal returns $\vartheta_F \equiv \frac{F'}{F}$ measures the absolute speed of satiation and given by*

$$|\eta_{\vartheta_F}| = -\frac{\vartheta'_F x}{\vartheta_F} = \sigma_F + \eta_F$$

where $\sigma_F \equiv -\eta_{F'} = -\frac{F''x}{F'}$ > 0 measures the concavity of F . Moreover, the elasticity of ϑ'_F measures the local acceleration of this satiation rate:

$$|\eta_{\vartheta'_F}| = -\frac{\vartheta''_F x}{\vartheta'_F} = \frac{\rho_F \sigma_F + \eta_F (3\sigma_F + 2\eta_F)}{\sigma_F + \eta_F}$$

where $\rho_F \equiv -\eta_{F''} = -\frac{F'''x}{F''}$ > 0 measures the speed of the concavity change of F .

Based on these definitions and expressions, the satiation indices can be written as $S_F = \frac{\sigma_F}{\rho_F}$ and $S_F^{\text{rel}} = \frac{|\eta_{\vartheta_F}|}{|\eta_{\vartheta'_F}|}$.

Example 1: Isoelastic Function. Consider $H(h) = \bar{H}h^\xi$ where $\bar{H} > 0$ and $\xi \in (0,1)$. In this case, $\eta_H = \xi$, $\sigma_H = 1 - \xi$ and $\rho_H = 2 - \xi$. Then, using the definition of satiation indices and Lemma B.1 yields $S_H(h) = \frac{1-\xi}{2-\xi}$ and $S_H^{\text{rel}}(h) = 1/2$.

Example 2: Utility Function. Consider the utility function $U(c) = \bar{u} + \frac{c^{1-\Sigma}}{1-\Sigma}$ with $\bar{u} > 0$ and $\Sigma > 1$. Note that $U(c) \geq 0$ if and only if $c > c_{\min} = [\bar{u}(\Sigma - 1)]^{\frac{1}{1-\Sigma}}$. We can show that $\eta_U(c) = \frac{1}{\bar{u}c^{\Sigma-1} + \frac{1}{1-\Sigma}}$, $\sigma_U = \Sigma$, and $\rho_U = \Sigma + 1$. Note $\eta_U(c)$ is decreasing in c with $\lim_{c \rightarrow c_{\min}} \eta_U(c) = \infty$ and $\lim_{c \rightarrow \infty} \eta_U(c) = 0$. Note that the satiation rate for U is

$$S_U(c) = \frac{\Sigma}{\Sigma + 1}.$$

Moreover, using Lemma B.1 yields $|\eta_{\vartheta_U}| = \Sigma + \eta_U$ and $|\eta_{\vartheta'_U}| = \Sigma + 2\eta_U + \frac{\Sigma}{\Sigma + \eta_U}$. As a result, the relative satiation rate for U is

$$S_U^{\text{rel}}(c) = \frac{(\Sigma + \eta_U(c))^2}{\Sigma(\Sigma + 1) + \eta_U(c)(3\Sigma + 2\eta_U(c))}$$

Based on the properties of $\eta_U(c)$, we can conclude that $\lim_{c \rightarrow c_{\min}} S_U^{\text{rel}}(c) = \frac{1}{2}$ and $\lim_{c \rightarrow \infty} S_U^{\text{rel}}(c) = \frac{\Sigma}{\Sigma + 1}$.²² Moreover, $S_U^{\text{rel}}(c)$ is U-shaped: it decreases up to certain level of consumption, but it rises beyond this point. In general, this increasing region prevails.

²²This characterization is also valid for function $U(x) = \bar{u} + \log(c)$ using $\eta_U(c) = \frac{1}{\bar{u} + \log(c)}$ and $\Sigma = 1$.

Example 3: Survival Functions. I also characterize the satiation rates for different survival rate functions. For all these functions, I use the same function linking the health stock h with health investments h : $H(h) = \bar{H}h^\xi$. I then specify a survival rate as a function of health stock $\tilde{\Psi}(H)$ and study the properties of the composed function $\Psi(h) = \tilde{\Psi}(H(h))$. Using this composite function, the elasticities of interest are $\eta_\Psi = \eta_{\tilde{\Psi}} \cdot \eta_H$, $\sigma_\Psi = \sigma_H + \sigma_{\tilde{\Psi}}\eta_H$ and

$$\rho_\Psi \cdot \sigma_\Psi = \rho_H \cdot \sigma_H + 3 \cdot \sigma_{\tilde{\Psi}} \cdot \sigma_H \cdot \eta_H + \rho_{\tilde{\Psi}} \cdot \sigma_{\tilde{\Psi}} \cdot \eta_H^2.$$

Linear Survival Function $\tilde{\Psi}$. Consider the simplest functional form $\tilde{\Psi}(H) = H$. In this case, $\eta_{\tilde{\Psi}} = \rho_{\tilde{\Psi}} = 1$, $\sigma_{\tilde{\Psi}} = 0$ and, as a result, $\eta_\Psi = \xi$, $\sigma_\Psi = 1 - \xi$, and $\rho_\Psi = 2 - \xi$. Thus, the relative satiation rate for Ψ is

$$S_\Psi^{\text{rel}}(h) = \frac{(1 - \xi + \xi)^2}{(1 - \xi)(2 - \xi) + \xi(3(1 - \xi) + 2\xi)} = \frac{1}{2}.$$

Based on this characterization, we can conclude that the elasticity of Ψ and the ratio $\phi = \frac{\eta_\Psi}{\sigma_\Psi + \eta_\Psi}$ are constants and equal to ξ .

Mortality as Inverse of Health. Consider the following survival rate function $\tilde{\Psi}(H) = 1 - \frac{1}{H}$. Taking derivative yields $\eta_{\tilde{\Psi}} = \frac{1}{H-1}$ and $\sigma_{\tilde{\Psi}} = \rho_{\tilde{\Psi}} = 1$ and, as a result,

$$\eta_\Psi(h) = \frac{\xi}{H(h) - 1}, \quad \sigma_\Psi = 1, \quad \text{and} \quad \rho_\Psi = 2 - \xi^2.$$

Let h_{\min} be the level of health investment such that $H(h_{\min}) = 1$. As a result, $\lim_{h \rightarrow h_{\min}} \eta_\Psi(h) = \infty$ and $\lim_{h \rightarrow \infty} \eta_\Psi(h) = 0$. Moreover, the ratio

$$\phi(h) = \frac{\eta_\Psi(h)}{\sigma_\Psi(h) + \eta_\Psi(h)} = \frac{\xi}{\xi + H(h) - 1}$$

has the following limit behavior: $\lim_{h \rightarrow h_{\min}} \phi(h) = 1$ and $\lim_{h \rightarrow \infty} \phi(h) = 0$. The satiation rate for Ψ is

$$S_\Psi^{\text{rel}}(h) = \frac{(1 + \eta_\Psi(h))^2}{2 - \xi^2 + \eta_\Psi(h)(3 + 2\eta_\Psi(h))}.$$

Thus, we can conclude that $\lim_{h \rightarrow h_{\min}} S_\Psi^{\text{rel}}(h) = \frac{1}{2}$ and $\lim_{h \rightarrow \infty} S_\Psi^{\text{rel}}(h) = \frac{1}{2 - \xi^2}$.

Logistic Function $\tilde{\Psi}$. Consider the logistic survival rate, i.e., $\tilde{\Psi}(H) = \frac{\exp(\bar{\Psi} + H)}{1 + \exp(\bar{\Psi} + H)}$. Taking derivative yields $\frac{\eta_{\tilde{\Psi}}}{H} = 1 - \tilde{\Psi}$, $\frac{\sigma_{\tilde{\Psi}}}{H} = 2\tilde{\Psi} - 1$, and

$$\frac{\rho_{\tilde{\Psi}} \sigma_{\tilde{\Psi}}}{H H} = \left(\frac{\sigma_{\tilde{\Psi}}}{H} \right)^2 - 2\tilde{\Psi} \frac{\eta_{\tilde{\Psi}}}{H}.$$

Moreover, $\frac{\eta_\Psi}{\xi H} = \frac{\eta_{\tilde{\Psi}}}{H}$, $\frac{\sigma_\Psi}{\xi H} = \frac{\sigma_H}{\xi H} + \frac{\sigma_{\tilde{\Psi}}}{H}$, and

$$\frac{\rho_\Psi \sigma_\Psi}{\xi H \xi H} + 3 \frac{\sigma_\Psi \eta_\Psi}{\xi H \xi H} + 2 \left(\frac{\eta_\Psi}{\xi H} \right)^2 = \frac{\rho_H \sigma_H}{\xi H \xi H} + 3\tilde{\Psi} \frac{\sigma_H}{\xi H} + \tilde{\Psi}(2\tilde{\Psi} - 1).$$

Based on this characterization, we conclude that the elasticity $\eta_\Psi = \zeta H(h)[1 - \Psi(h)]$, $\sigma_\Psi = \sigma_H + \zeta H(h)[2\Psi(h) - 1]$ and the ratio

$$\phi(h) = \frac{\eta_\Psi(h)}{\sigma_\Psi(h) + \eta_\Psi(h)} = \frac{\zeta H(h)[1 - \Psi(h)]}{\sigma_H + \zeta H(h)\Psi(h)}.$$

Thus, the limit behavior is given by $\lim_{h \rightarrow 0} \phi(h) = \lim_{h \rightarrow \infty} \phi(h) = 0$. Note that Lemma B.1 is valid under these rescaled definitions of elasticities. Then, the satiation rate for Ψ is

$$S_\Psi^{\text{rel}}(h) = \frac{\left(\frac{\sigma_H}{\zeta H(h)} + \Psi(h)\right)^2}{\frac{\rho_H}{\zeta H(h)} \frac{\sigma_H}{\zeta H(h)} + \Psi(h) \left(2\Psi(h) - 1 + 3\frac{\sigma_H}{\zeta H(h)}\right)}.$$

Although this is a complicated expression, we can characterize the satiation index for Ψ on the limits $h \rightarrow 0$ and $h \rightarrow \infty$. First, note that $\frac{\rho_H}{\sigma_H} = \frac{2-\zeta}{1-\zeta}$ and the following limits holds: $\lim_{h \rightarrow 0} \frac{\Psi(h)}{\sigma_H} \zeta H(h) = 0$ and $\lim_{h \rightarrow \infty} \frac{\sigma_H}{\zeta H(h)} = \lim_{h \rightarrow \infty} \frac{\rho_H}{\zeta H(h)} \frac{\sigma_H}{\zeta H(h)} = 0$. Next, we can show that

$$\lim_{h \rightarrow 0} S_\Psi^{\text{rel}}(h) = \lim_{h \rightarrow 0} \frac{\left(1 + \frac{\Psi(h)}{\sigma_H} \zeta H(h)\right)^2}{\frac{2-\zeta}{1-\zeta} + \zeta H(h) \frac{\Psi(h)}{\sigma_H} \left(\zeta H(h) \frac{2\Psi(h)-1}{\sigma_H} + 3\right)} = \frac{1-\zeta}{2-\zeta}$$

and $\lim_{h \rightarrow \infty} S_\Psi^{\text{rel}}(h) = 1$. Thus, the relative satiation index is increasing and eventually concave converging to 1 while h grows.

B.2 A Full Derivation of the Microeconomic Setting

Consider the following more general optimization problem:

$$\max_{a, h} u(y - a - h) + \Psi(h) \cdot U(C(a, h), h) + [1 - \Psi(h)] \cdot \mathcal{U}(a) \quad (\text{B.1})$$

where $U(c, h)$ is a health dependent utility function for the second period, $C(a, h)$ is a consumption function for the elderly, and \mathcal{U} is the utility from bequest. The consumption function $C(a, h)$ mainly captures the type of assets available for financial investments and other pecuniary benefits from having better health (e.g., lower long-term care needs or higher labor income). For instance, it may be specified by $C(a, h) = Ra$ in a world where there are not pecuniary benefits of better health and assets are invested in riskless bonds.

As in the main text, I rewrite the problem as a two-stage problem. The intertemporal problem is the same as the one studied in the main text, but with properties of V inherited from the new portfolio problem:

$$V(\omega) \equiv \max_{\alpha, \lambda} \Psi(\lambda) \cdot U(C(\alpha, \lambda), \lambda) + [1 - \Psi(\lambda)] \cdot \mathcal{U}(\alpha) \quad \text{subject to: } \alpha + \lambda = \omega. \quad (\text{B.2})$$

Again, the optimal composition of wealth should equalize the marginal return of health and assets. The marginal return of assets is now given by $\Psi \partial_\alpha C \partial_c U + (1 - \Psi) \mathcal{U}'$, while the marginal return on health is $\Psi(\partial_\lambda C \partial_c U + \partial_\lambda U) + \Psi'(U - \mathcal{U})$. Rearranging this optimal condition yields

$$\frac{\Psi'}{\Psi} \cdot \left(1 - \frac{\mathcal{U}}{U}\right) = (\partial_\alpha C - \partial_\lambda C) \cdot \frac{\partial_c U}{U} - \frac{\partial_\lambda U}{U} + \frac{1 - \Psi}{\Psi} \cdot \frac{\mathcal{U}'}{U} \cdot \frac{U}{U}. \quad (\text{B.3})$$

B.2.1 On the Curvature of Asset Function

Intertemporal Saving. Consider (2) and study the curvature of $\omega(y)$. The following lemma provides further algebraic results than in Lemma 1 used in the main text.

Lemma B.2. *Let $\omega(y)$ be the optimal total investment policy for problem (2). Then, the concavity measure for ω is*

$$\sigma_\omega = (1 - \omega') \cdot \sigma_V \cdot \eta_\omega \cdot (1/S_u - 1/S_V) \quad (\text{B.4})$$

where all the objects are evaluated at ω , $\alpha(\omega)$ and $\lambda(\omega)$.

Proof. See Appendix E.1. □

Portfolio Problem. Below I use (B.3) to study the effects of additional channels of health investment and the role of bequest motives. However, in the following derivation I abstract from these additional channels. As a result, the analysis reduces to the model studied in the main text, with the only difference being that I allow for potentially different utility functions in the first and second periods. The key results are summarized in the following proposition.

Lemma B.3. *The satiation index of the continuation value V is*

$$S_V = S_U \cdot \frac{\left(1 - \phi \frac{|\eta_{\theta_U}|}{\sigma_U}\right)^2}{1 + \frac{1}{\rho_U} \left[\frac{|\eta_{\theta_U}|}{\sigma_U} \frac{\sigma_\alpha}{\eta_\alpha} - \eta_\Psi \frac{\eta_\lambda}{\eta_\alpha} \left(2 + \frac{\sigma_\Psi}{\sigma_U} \frac{\eta_\lambda}{\eta_\alpha}\right) \right]} \quad (\text{B.5})$$

where $\phi \equiv \frac{\eta_\Psi}{\sigma_\Psi + \eta_\Psi}$. Moreover, the concavity for $\alpha(\omega)$ is

$$\sigma_\alpha = (1 - \alpha') \cdot \eta_\alpha \cdot |\eta_{\theta_U}| \cdot \left(1/S_\Psi^{rel} - 1/S_U^{rel}\right). \quad (\text{B.6})$$

Note that all the elasticities are evaluated at ω , $\alpha(\omega)$ and $\lambda(\omega)$.

Proof. See Appendix E.2. □

Satiation Index of the Continuation Function. Endogenous longevity affects the satiation index of the value function V . Further progress is possible under a proper parameterization of U and Ψ .

Proposition B.1. *Suppose functions U and Ψ are given by (6) and (7). Then, the satiation rate S_V is increasing in ω with $\lim_{\omega \rightarrow 0} S_V(\omega) < S_U$ and $\lim_{\omega \rightarrow \infty} S_V(\omega) > S_U$.*

Based on the results in Proposition B.1 and Lemma B.2, it is straightforward to show the following proposition.

Proposition B.2. *Suppose u and U are both given by the same (6) and Ψ by (7). Without endogenous longevity, optimal policy ω is linear in income and, thus, $\sigma_\omega = 0$. When the individuals can control their longevity through health investments, there exists a level of income \hat{y}_ω such that the policy ω is convex, $\sigma_\omega < 0$, for $y \in (0, \hat{y}_\omega]$ and concave, $\sigma_\omega > 0$ otherwise.*

Concavity of the Financial Investment Function. I show that the concavity of α , σ_α , depends primarily on the relative satiation indices. We can make further progress on this regard by using the parameterization suggested in the main text.

Proposition B.3. *Suppose functions U and Ψ are given by (6) and (7). The index $S_U^{rel}(c)$ rises with consumption: it takes value $1/2$ when consumption is near c_{min} , and it gradually increases toward $\frac{\Sigma}{\Sigma+1}$ as consumption becomes very large. Similarly, the index $S_\Psi^{rel}(h)$ increases with health investment, approaching $\frac{1-\xi}{2-\xi}$ as h becomes very small, and it gradually rises toward 1 as h becomes very large. As a result, there exists a threshold $\hat{\omega} > 0$ such that $\alpha(\omega)$ is concave for $\omega \in (0, \hat{\omega}]$ and convex for $\omega > \hat{\omega}$.*

Proof. See Appendix E.4. □

Overall Asset Policy, $a(y)$. Due to endogenous longevity, the curvature of the optimal policy as a function of income changes. The concavity of a , given by $\sigma_a = \sigma_\alpha \frac{y}{\omega} + \sigma_\omega$, depends on the relative importance of the concavity of ω and α which are characterized by (B.4) and (B.6), respectively. Although a complete characterization of this curvature is complicated, I know show crucial properties of the curvature of a under a special parameterization.

Proposition B.4. *Suppose u and U are both given by the same (6) and Ψ by (7). It turns out that there exists a threshold \hat{y} such that the asset policy $a(y)$ solving problem (1) is concave for $y \in (0, \hat{y}]$ and convex otherwise.*

Robustness. So far I have characterized key properties of a when the second-stage policy function α is determined by $\frac{\Psi'}{\Psi} = \partial_\alpha C \cdot \frac{\partial_c U}{U}$ with $\partial_\alpha C$ being constant. That is, in my analysis, I abstracted from (i) pecuniary (captured by $\partial_h U$) and nonpecuniary benefits (captured by $\partial_\lambda C \partial_c U$) of health, and (ii) altruism in the form of bequest motives (captured by U'). Now, I use my previous findings to provide arguments about how these other determinants of financial and health investments affect my main results.

B.2.2 Elasticities of Savings

Another aspect altered by the endogenous longevity channel on the individual decisions is the elasticity of individual savings with respect to other aspects of their circumstances. In particular, I am interest is in movements in interest rate and survival rate parameters. Let x denote an exogenous movement in either interest rate or a paramter in the survival rate function, then the elasticity of savings with respect to x is

$$\frac{\partial a}{\partial x} \frac{x}{a} = \frac{\partial \alpha}{\partial x} \frac{x}{\alpha} + \eta_\alpha \cdot \frac{\partial \omega}{\partial x} \frac{x}{\omega} \quad \text{where} \quad \frac{\partial \omega}{\partial x} \frac{x}{\omega} \equiv \frac{\frac{\partial V'}{\partial x} \frac{x}{V'}}{\frac{\omega}{c_y} \sigma_u + \sigma_V}.$$

Interest rate elasticity. From the optimality conditions for problems (4) and (2), we have:

$$\frac{\partial \alpha}{\partial R} \frac{R}{\alpha} = \frac{1 - |\eta_{\theta_U}|}{|\eta_{\theta_U}| + \frac{\alpha}{\lambda} |\eta_{\theta_\Psi}|} \quad \text{and} \quad \frac{\partial V'}{\partial R} \frac{R}{V'} = 1 - \sigma_U.$$

$$\frac{\partial a}{\partial R} \frac{R}{a} = \frac{1 - |\eta_{\theta_U}|}{|\eta_{\theta_U}| + \frac{\alpha}{\lambda} |\eta_{\theta_\Psi}|} + \eta_\alpha \cdot \frac{1 - \sigma_U}{\frac{\omega}{c_y} \sigma_U + \sigma_V}$$

Survival rate elasticity. Similarly, the elasticity with respect to survival rates.

$$\frac{\partial \alpha}{\partial x} \frac{x}{\alpha} = -\frac{\frac{\partial \theta_\Psi}{\partial x} \frac{x}{\theta_\Psi}}{\eta_U + \frac{\alpha}{\lambda} \eta_\Psi} \quad \text{and} \quad \frac{\partial V'}{\partial x} \frac{x}{V'} = \frac{\partial \Psi}{\partial x} \frac{x}{\Psi}.$$

$$\frac{\partial a}{\partial x} \frac{x}{a} = -\frac{\frac{\partial \theta_\Psi}{\partial x} \frac{x}{\theta_\Psi}}{\eta_U + \frac{\alpha}{\lambda} \eta_\Psi} + \eta_\alpha \cdot \frac{\frac{\partial \Psi}{\partial x} \frac{x}{\Psi}}{\frac{\omega}{c_y} \sigma_U + \sigma_V}$$

From the optimality conditions for problems (4) and (2), we have:

1. Elasticity with respect to interest rate:

$$\frac{\partial \alpha}{\partial x} \frac{x}{\alpha} = \frac{1 - \kappa_U}{\eta_U + \frac{\alpha}{\lambda} \eta_\Psi} < 0 \quad \text{and} \quad \frac{\partial V'}{\partial x} \frac{x}{V'} = 1 - \kappa_U \frac{\partial \alpha}{\partial x} \frac{x}{\alpha}.$$

2. Elasticity with respect to survival:

$$\frac{\partial \alpha}{\partial x} \frac{x}{\alpha} = -\frac{\frac{\partial \theta_\Psi}{\partial x} \frac{x}{\theta_\Psi}}{\eta_U + \frac{\alpha}{\lambda} \eta_\Psi} \quad \text{and} \quad \frac{\partial V'}{\partial x} \frac{x}{V'} = \frac{\partial \Psi}{\partial x} \frac{x}{\Psi} - \kappa_U \frac{\partial \alpha}{\partial x} \frac{x}{\alpha}.$$

B.3 Laissez-Faire Equilibrium

In this section, suppose that z can take several values and $\bar{x} = \mathbb{E}_z[x(z)]$ denotes the average over z .

Individual Decisions. Given productivity z and prices, the individual household solves

$$\max_{a, h} u(c_{y,t}) + \Psi_{t+1}(h_t) \cdot U(c_{o,t+1}, h_t) + [1 - \Psi_{t+1}(h_t)] \cdot U(a_t)$$

subject to:

$$c_{y,t} + h_t + a_t = w_t \cdot z + \mathbf{b}q_t \tag{B.7}$$

$$c_{o,t+1} + M_{t+1}(h_t) = R_{t+1} \cdot a_t. \tag{B.8}$$

Saving decisions obey the standard Euler equation but with an expected return is affected by their survival rates:

$$u'(c_{y,t}) = R_{t+1} \cdot \Psi_{t+1}(h_t) \cdot U_c(c_{o,t+1}, h_t) + [1 - \Psi_{t+1}(h_t)] \cdot U'(a_t). \tag{B.9}$$

The optimal health investment balances the current marginal cost of higher health expenses with the marginal benefits of health expenses, in turn, determined by the value of future life and the changes in medical expenses:

$$u'(c_{y,t}) = \Psi'_{t+1}(h_t) [U(c_{o,t+1}, h_t) - U(a_t)] + \Psi_{t+1}(h_t) [U_h(c_{o,t+1}, h_t) - M'_{t+1}(h_t)U_c(c_{o,t+1}, h_t)]. \quad (\text{B.10})$$

Aggregation. The aggregate consumption for all individuals alived at date t is

$$N_{y,t}(\bar{c}_{y,t} + \bar{h}_t) = N_{y,t}w_t + N_{y,t}\text{bq}_t - N_{y,t}\bar{a}_t$$

$$N_{o,t}(\bar{c}_{o,t} + \bar{M}_t) = R_t N_{y,t-1} \mathbb{E}_z[\psi_t(z)a_{t-1}(z)]$$

Combining these two equations with the bequest equation (12), the aggregate demand of final goods should satisfy

$$N_{y,t}(\bar{c}_{y,t} + \bar{h}_t) + N_{o,t}(\bar{c}_{o,t} + \bar{M}_t) = N_{y,t}w_t + R_t N_{y,t-1}\bar{a}_{t-1} - N_{y,t}\bar{a}_t.$$

Equilibrium. I next define the competitive equilibrium by using per-labor-size variables, i.e., using the normalized model. In particular, I define the capital-labor-size ratio as $k_t \equiv K_t/N_{y,t}$ and $f_t(k) \equiv F_t(k, 1)$ is the production function under the average labor.

Definition 3. Given initial stocks $\{h_{-1}, k_{-1}\}$, a sequence of demographic and health technologies $\{n_t, \Psi_t, M_t\}_{t \geq 0}$, a sequence of productivity technologies $\{F_t, \Pi_t\}_{t \geq 0}$, and sequence of bequest rules $\{\text{bq}_t\}_{t \geq 0}$, a competitive equilibrium is an allocation $\{h_t, a_t, c_{y,t}, c_{o,t}, k_t\}_{t \geq 0}$, demographic structure $\{d_t\}_{t \geq 0}$, and price sequence $\{w_t, r_t, R_t\}_{t \geq 0}$ such that: (i) Given $\{w_t, R_t\}_{t \geq 0}$ and $\{n_t, \Psi_t, M_t\}_{t \geq 0}$, household's decision over $\{h_t, a_t, c_{y,t}, c_{o,t+1}\}_{t \geq 0}$ satisfies (B.7), (B.8), (B.9), and (B.10) where bequests satisfy (12) and $R_t = 1 - \delta + r_t$; (ii) the first elderly population, consumption is given by

$$c_{o,t+1}(z) = R_{t+1}a_t(z) - M_{t+1}(h_t(z));$$

(iii) Given $\{w_t, r_t\}_{t \geq 0}$ and $\{F_t\}_{t \geq 0}$, the representative firm acts optimally: $r_t = f'_t(k_t)$ and $w_t = f_t(k_t) - k_t f'_t(k_t)$; (iv) Demographics are given by (10); and (iv) Markets clear: For all $t \geq 0$, (i) Capital market, $(1 + n_t)k_t = \bar{a}_{t-1}$, and (ii) Final good's market:

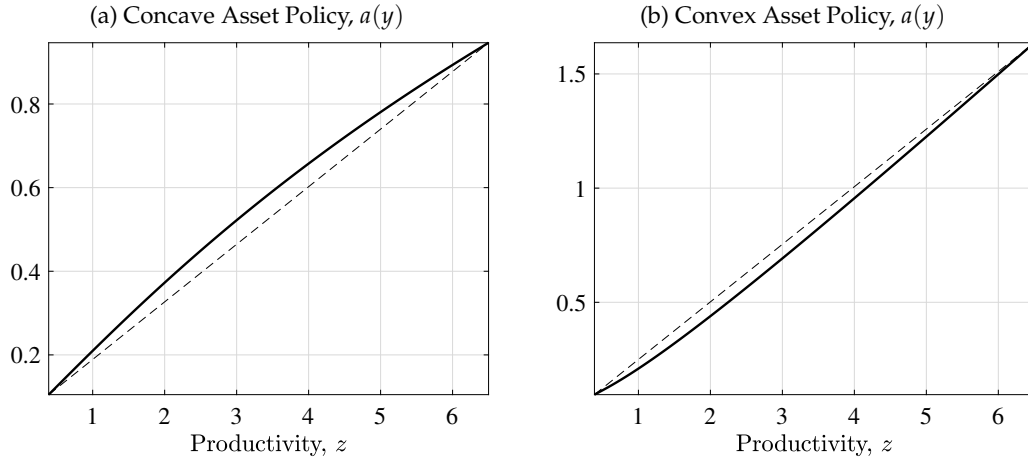
$$f_t(k_t) + (1 - \delta)k_t = \bar{c}_{y,t} + \bar{h}_t + d_t(\bar{c}_{o,t} + \bar{M}_t) + (1 + n_{t+1})k_{t+1}.$$

It is worth noting that the aggregate demographic structure affects the equilibrium only through its impact on the aggregate resource constraint.

B.4 General Equilibrium Effects

In the main text, I discuss the general equilibrium effects of increments in income inequality. In Figure B.1, I plot the asset policy functions associated with the regime where this is concave (Panel a) and convex (Panel b).

Figure B.1: Asset Policy in the Numerical Exercise

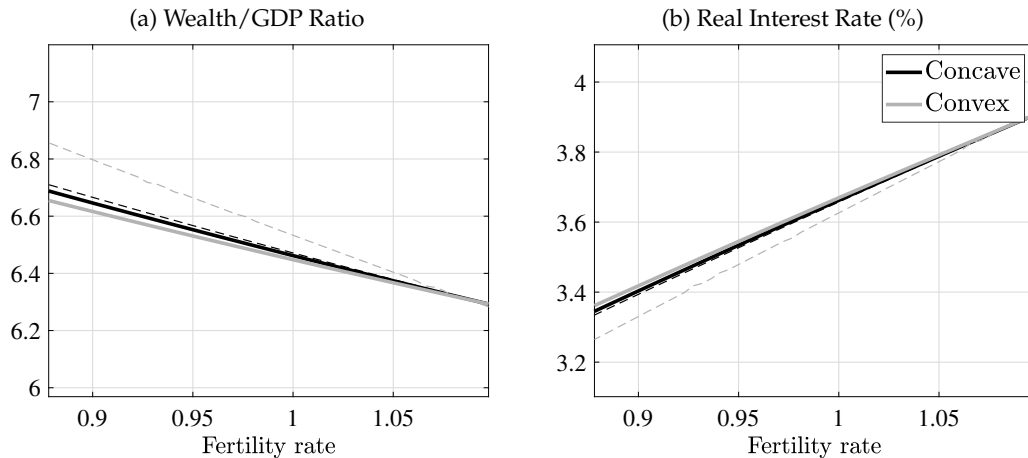


Note. In this plot $y(z) = w \cdot z + b_q$ for a given wage rate w and bequest transfers b_q .

Moreover, in this section, I briefly comment on the implications of the endogenous longevity for the general equilibrium effects of demographic changes, i.e., rise in life expectancy and decline in fertility rate.

Figure B.2 plots the general equilibrium effects for a decline in the fertility rate. Since the results are similar regardless of the curvature of the asset policy function, I only show the effects when this policy is convex. The key result is, relative to a model with exogenous longevity, real interest rates fall by less in the face of a declining fertility rate. The intuition is that the elasticity of assets with respect to interest rate movements is higher when longevity is endogenous.

Figure B.2: Equilibrium Effects of Changes in the Fertility Rate

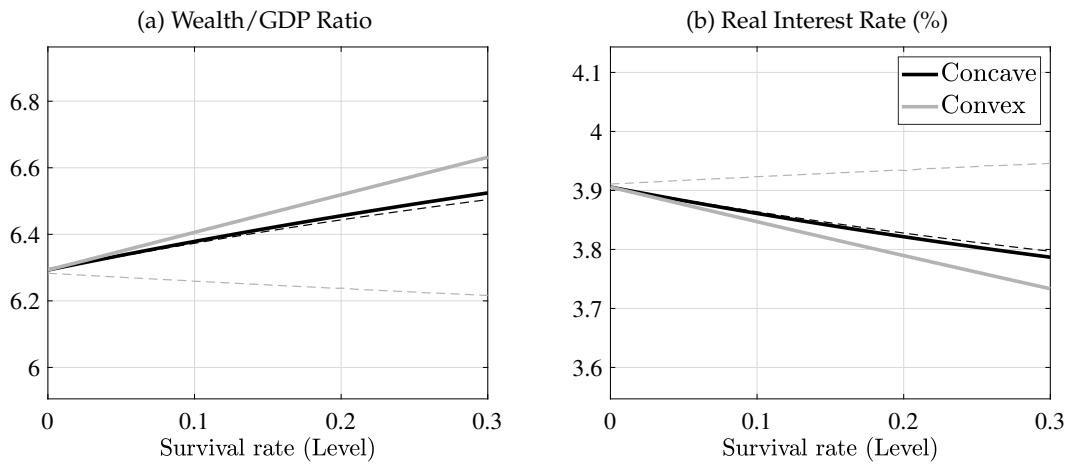


Note. Solid-black lines represent the model with endogenous longevity, while dashed-black lines model with exogenous longevity.

Figure B.3 plots the general equilibrium effects for a rise in the survival rate technology. In particular, this figure deploys the case where ψ_0 (level parameter) increases. As I discuss in the main text, under this type of technology changes, saving behavioral changes are stronger when longevity is

endogenous. As a result, the decline in the real interest rate associated with this exogenous movement is stronger.

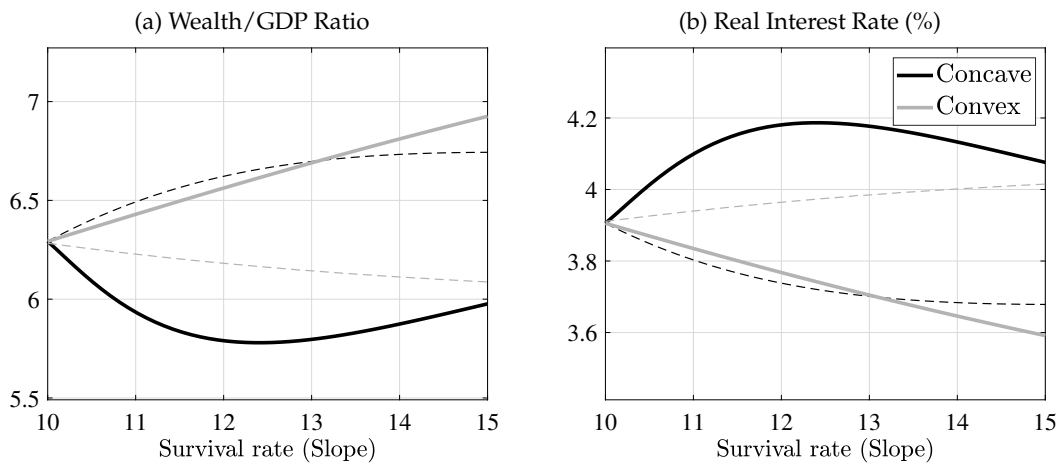
Figure B.3: Equilibrium Effects of Changes in the Survival Rate (Level)



Note. Solid-black lines represent the model with endogenous longevity, while dashed-black lines model with exogenous longevity.

Figure B.4 plots the general equilibrium effects for a rise in the survival rate when ψ_h (slope parameter) is the driver of this technology improvement. In the main text, I emphasize that the curvature of the asset policy is key.

Figure B.4: Equilibrium Effects of Changes in the Survival Rate (Slope)



Note. Solid-black lines represent the model with endogenous longevity, while dashed-black lines model with exogenous longevity.

B.5 Competitive Equilibrium with Government

Suppose the market economy is as in Section 3 and there is a government as described in Assumption 2. In a competitive equilibrium, households' optimal policies $\{h_t(z), a_t(z), c_{y,t}(z), c_{o,t+1}(z)\}$ solves

$$\begin{aligned} c_{y,t} + (1 + \tau_t^h)h_t + a_t^k + a_t^b + \tau_t(z) &= zw_t + \mathbf{bq}_t \\ c_{o,t+1} + M(h_t) &= (1 - \tau_{t+1}^k)R_{t+1}^k a_t^k + R_t^b a_t^b + s_{t+1}(z) \\ (1 + \tau_t^h)u'(c_{y,t}) &= U'(c_{o,t+1}) \left[\Psi'_{t+1}(h_t) \frac{U(c_{o,t+1})}{U'(c_{o,t+1})} - \Psi_{t+1}(h_t) M'(h_t) \right] \end{aligned}$$

where $R_t^k = 1 - \delta + r_t$. The optimal financial portfolio decision depends on the return on k and b :

$$u'(c_{y,t}) = \begin{cases} (1 - \tau_{t+1}^k)R_{t+1}^k \Psi_{t+1}(h_t) U'(c_{o,t+1}) & \text{If } (1 - \tau_{t+1}^k)R_{t+1}^k > R_t^b \\ R_{t+1}^b \Psi_{t+1}(h_t) U'(c_{o,t+1}) & \text{Otherwise} \end{cases}$$

Moreover, for the initial elderly

$$c_{o,0} + M(h_{-1}(z)) = (1 - \tau_0^k)R_0^k a_{-1}^k(z) + R_0^b a_{-1}^b(z) + s_0(z).$$

Firms maximize profits yielding to the following demand of factors

$$r_t = F_{k,t}(K_t/N_{y,t}, 1) \quad \text{and} \quad w_t = F_{l,t}(K_t/N_{y,t}, 1).$$

Government's policy instruments satisfy

$$\begin{aligned} B_{t+1}^g + N_{y,t} \left(\mathbb{E}_z[\tau_t(z)] + \tau_t^h \mathbb{E}_z[h_t(z)] + \mathbb{E}_z[a_t^b(z)] \right) + R_t^k K_t^g + N_{y,t-1} \tau_t^k R_t^k \mathbb{E}_z[a_{t-1}^k(z)] \\ = N_{o,t} \left(\mathbb{E}_z[s_t(z)] + R_t^b \mathbb{E}_z[a_{t-1}^b(z)] \right) + K_{t+1}^g + R_t B_t^g \end{aligned}$$

where $K_{g,t}$ denotes the total physical holdings of the government. Finally, markets clear

$$B_{t+1}^g + K_{t+1} = N_{y,t} \mathbb{E}_z[a_{t+1}^k(z)] + K_{t+1}^g.$$

Combining the aggregate constraints yields

$$F_t(K_t, N_{y,t}) + (1 - \delta)K_t = N_{y,t}(\bar{c}_{y,t} + \bar{h}_t) + N_{o,t}(c_{o,t} + \bar{M}_t) + K_{t+1}.$$

B.5.1 Government's Optimal Policy Implementation

Implementation with Annuities. Under the policy described in Proposition 8, the government's capitalization is described by $K_{t+1}^g = N_{y,t} \mathbb{E}_z[a_t^b(z)]$ and $R_{t+1}^k K_{t+1}^g = R_{t+1}^b N_{o,t+1} \mathbb{E}_z[a_t^b(z)]$ yielding

$R_{t+1}^b = \frac{R_{t+1}^k}{\Psi_{t+1}} > R_{t+1}^k$. Then, the equilibrium conditions become

$$c_{y,t} + (1 + \tau_t^h)h_t + a_t + \bar{\tau}_t = \bar{z}_t w_t + \mathbf{bq}_t$$

$$\begin{aligned}
c_{o,t+1} + M_{t+1}(h_t) &= \frac{R_{t+1}^k}{\Psi_{t+1}(h_t)} a_t + \bar{s}_{t+1} \\
u'(c_{y,t}) &= R_{t+1}^k U'(c_{o,t+1}) \\
(1 + \tau_t^h) u'(c_{y,t}) &= U'(c_{o,t+1}) \left[\Psi'_{t+1}(h_t) \frac{U(c_{o,t+1})}{U'(c_{o,t+1}^o)} - \Psi_{t+1}(h_t) M'(h_t) \right]
\end{aligned}$$

and clearing market $(1 + n_t)k_t = a_{t-1}$, where $r_t = F_{k,t}(k_t, 1)$, $w_t = F_{l,t}(k_t, 1)$, and $\text{bq}_t = \begin{cases} (1 - \Psi_{-1})R_0^k k_0 & t = 0 \\ 0 & t > 0 \end{cases}$. Moreover, for the initial elderly $c_{o,0} + M(h_{-1}) = (1 + n_0)R_0^k k_{-1} + \bar{s}_0$. Given any desired allocation $\{h_t^*, c_{y,t}^*, c_{o,t}^*, k_{t+1}^*\}_{t \geq 0}$, the optimal transfer policy solves

$$\bar{s}_t = \begin{cases} c_{o,0}^* + M(h_{-1}) - (1 + n_0)R_0^k k_{-1} & t = 0 \\ c_{o,t}^* + M(h_{t-1}^*) - \frac{1}{d_t^*} R_t^* k_{t-1}^* & t \geq 1 \end{cases}.$$

Moreover, from the government's budget constraint: $\bar{\tau}_t = d_t \bar{s}_t - \tau_t^h h_t$.

Implementation with Capital Subsidies. Consider now the policy described in Lemma 2, the equilibrium conditions become

$$\begin{aligned}
c_{y,t} + (1 + \tau_t^h) h_t + a_t + \bar{\tau}_t &= \bar{z}_t w_t + \text{bq}_t \\
c_{o,t+1} + M(h_t) &= (1 - \tau_{t+1}^k) R_{t+1}^k a_t + \bar{s}_{t+1} \\
u'(c_{y,t}) &= (1 - \tau_{t+1}^k) R_{t+1}^k U'(c_{o,t+1}) \\
(1 + \tau_t^h) u'(c_{y,t}) &= U'(c_{o,t+1}) \left[\Psi'_{t+1}(h_t) \frac{U(c_{o,t+1})}{U'(c_{o,t+1})} - \Psi_{t+1}(h_t) M'(h_t) \right]
\end{aligned}$$

and $(1 + n_t)k_t = a_{t-1}$, where $r_t = F_{k,t}(k_t, 1)$, $w_t = F_{l,t}(k_t, 1)$, and $\text{bq}_t = (1 - \Psi_t)R_t^k k_{t-1}$. Moreover, for the initial elderly $c_{o,0} + M(h_{-1}) = (1 + n_0)R_0^k k_{-1} + \bar{s}_0$. Given any optimal allocation $\{h_t, c_{y,t}, c_{o,t}, k_{t+1}\}_{t \geq 0}$, the optimal transfer policy solves

$$\bar{s}_t = \begin{cases} c_{o,0}^* + M(h_{-1}) - (1 + n_0)R_0^k k_{-1} & t = 0 \\ c_{o,t}^* + M(h_{t-1}^*) - (1 - \tau_t^k)(1 + n_t)R_t^k k_{t-1} & t \geq 1 \end{cases}.$$

Moreover, from the government's budget constraint: $\bar{\tau}_t + \tau_t^h h_t + \tau_t^k R_t^k k_t = d_t \bar{s}_t$.

B.5.2 Equilibrium Conditions for Section 4.3.1

In the economy described in Section 4.3.1, suppose we are given a stationary trajectory $\{h_t, c_{y,t}, c_{o,t}, k_t\}$ that satisfies

$$\begin{aligned}
u'(c_{y,t}) &= \beta R_{t+1}^k \Psi_{t+1}(h_t) U'(c_{o,t+1}) \\
\bar{z}_t f_t(k_t) + (1 - \delta)k_t &= c_{y,t} + h_t + d_t(c_{o,t} + M(h_{t-1})) + (1 + n_{t+1})k_{t+1}.
\end{aligned}$$

From the individual health's decision we can infer the health tax rate

$$1 + \tau_t^h = \frac{U'(c_{o,t+1})}{u'(c_{y,t})} \left[\Psi'_{t+1}(h_t) \frac{U(c_{o,t+1})}{U'(c_{o,t+1})} - \Psi_{t+1}(h_t) M'(h_t) \right].$$

Given any public debt holdings, the private capital holdings are $a_{t-1} = (1 + n_t)(b_{g,t} + k_t)$ for all $t \geq 0$. From the individual's budget constraints and $bq_t = (1 - \Psi_t)R_t^k k_{t-1}$, we can compute $\bar{\tau}_t$ and \bar{s}_t . Since agent's budget constraint and aggregate resource constraints are verified, the government's budget constraint is. Hence, equations (20) and (24) represents the implementable allocations for a government as in Section 4.3.1.

B.5.3 Equilibrium Conditions for Section 4.3.2

First, consider the case where there are not annuities at all. In this case, for all $z \sim \Pi_t(z)$ the solution for generation t decisions are pinned down by:

$$\begin{aligned} c_{y,t} + (1 + \tau_t^h)h_t + a_t + \bar{\tau}_t &= z\omega_t + bq_t \\ c_{o,t+1} + M(h_t) &= R_{t+1}a_t + \bar{s}_{t+1} \\ u'(c_{y,t}) &= \beta R_{t+1} \Psi_{t+1}(h_t) U'(c_{o,t+1}) \\ (1 + \tau_t^h)u'(c_{y,t}) &= \beta U'(c_{o,t+1}) \left[\Psi'_{t+1}(h_t) \frac{U(c_{o,t+1})}{U'(c_{o,t+1})} - \Psi_{t+1}(h_t) M'(h_t) \right]. \end{aligned}$$

Consider now that we are given an allocation $\{c_{y,t}(z), c_{o,t}(z), h_t(z), k_{t+1}\}_{t \geq 0}$ satisfying

$$\begin{aligned} \text{cte} &= \frac{\Psi'_{t+1}(h_t(z))}{\Psi_{t+1}(h_t(z))} \frac{U(c_{o,t+1}(z))}{U'(c_{o,t+1}(z))} - M'(h_t(z)) \\ u'(c_{y,t}(z)) &= \beta R_{t+1} \Psi_{t+1}(h_t(z)) U'(c_{o,t+1}(z)) \\ \bar{z}_t f_t(k_t/\bar{z}_t) - (1 - \delta)k_t &= \int (c_{y,t}(z) + h_t(z)) d\Pi_t \\ &+ \frac{1}{1 + n_t} \int (c_{o,t}(z) \Psi_t(h_{t-1}(z)) + M(h_{t-1}(z)) \Psi_t(h_{t-1}(z))) d\Pi_{t-1} + (1 + n_{t+1})k_{t+1}. \end{aligned}$$

We can find the proper tax instruments $\bar{\tau}_t$, \bar{s}_t and τ_t^h that decentralizes this allocation. Note that given $\{c_{y,t}(z), c_{o,t+1}(z), h_t(z)\}$, we can find a health tax rate τ_t^h that rationalizes this decision. Moreover, aggregating the individual allocations, we can find $\{\bar{\tau}_t, \bar{s}_t\}$ that verifies the government constraint. Finally, given allocations and lump-sum transfers, we can compute the capital distribution, $k_{b,t}(z)$, rationalizing this allocation. Thus, the above-described set of conditions are the implementable conditions for a government in this economy.

B.6 Numerical Solution

Although the below-described algorithms work under more general calibrations, the numerical illustrations in Section 3 are based on the following parameterization.

Assumption 3. Our benchmark parameterization assumes (i) utility functions are $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ and $U(c) = \bar{U} + \frac{c^{1-\Sigma}}{1-\Sigma}$ with $\bar{U} \geq 0$ and $\sigma, \Sigma > 0$; (ii) the production function is $F(K, L) = ZK^\alpha L^{1-\alpha}$ with $Z > 0$; (iii) the medical expenses function is $M(h) = \bar{M} \exp(-\zeta_m h)$; and (iv) the survival rate is given by one of the functional forms studied in Appendix B.1.

C Omitted Derivations for Section 4

In this section, I formalize the results discussed in Section 4 under a slightly more general version of the model. In particular, I assume that the second-period utility function U is potentially different than u in the first period and that the individuals should pay for a health-dependent long-term medical care when elderly, $M(h)$.

C.1 Derivations of Main Results

Suppose that the Pareto weights of the planner are $\{\Phi_{t-1}\}_{t \geq 0}$ satisfying $\sum_{t=0}^{\infty} \Phi_t < \infty$ for all $t \geq 0$. Given K_0 and $\{\mathcal{N}_{t-1}\}_{t \geq 0}$, the planner chooses $\{c_{y,t}^i, c_{o,t}^i, h_t^i\}_i, K_{t+1}\}_{t \geq 0}$ to maximize

$$\sum_{t=0}^{\infty} \frac{\Phi_t}{N_{y,t}} \int_{i \in \mathcal{N}_t} \left[u(c_{y,t}^i) + \Psi_{t+1}(h_t^i) U(c_{o,t+1}^i) \right] \partial i + \frac{\Phi_{-1}}{N_{y,-1}} \int_{i \in \mathcal{N}_{-1}} \Psi_0(h_{-1}^i) U(c_{o,0}^i) \partial i$$

subject to the resource constraint

$$F_t(K_t, N_{y,t}) + (1 - \delta)K_t = \int_{i \in \mathcal{N}_t} (c_{y,t}^i + h_t^i) \partial i + \int_{i \in \mathcal{N}_{t-1}} \Psi_t(h_{t-1}^i) [c_{o,t}^i + M_t(h_{t-1}^i)] \partial i + K_{t+1}$$

where \mathcal{N}_t is the set of individuals born at date t . Under concavity in u and U , first-order conditions are sufficient to characterize optimal paths in the standard settings. With the current endogenous-health setting, however, additional restrictions are required.

Assumption 4. The value of life as a ratio to consumption is larger than one, $\Theta(c) \equiv \frac{U(c)}{U'(c)c} \geq 1$, and the expected medical expense, $G(h) \equiv \Psi(h)M(h)$, is convex in h . Past investment decisions are uniform: $h_{-1}^i = h_{-1}$ for all $i \in \mathcal{N}_{-1}$.

Proposition C.1. Under Assumption 4, the planner allocates the same $(h_t, c_{y,t}, c_{o,t+1})$ for all agents in generation t . Moreover, the optimal planner's allocation is characterized by: (i) optimal intragenerational allocation on consumption and health investments:

$$u'(c_{y,t}) = (1 + \chi_{s,t+1}) R_{t+1} \Psi_{t+1}(h_t) U'(c_{o,t+1}) \quad (\text{C.1})$$

$$(1 + \chi_{h,t+1}) (1 + \chi_{s,t+1}) R_{t+1} = \frac{\Psi'_{t+1}(h_t) U(c_{o,t+1})}{\Psi_{t+1}(h_t) U'(c_{o,t+1})} - M'_{t+1}(h_t) \quad (\text{C.2})$$

where $\chi_{s,t+1} \equiv \frac{1}{\Psi_{t+1}} - 1$, $\chi_{h,t+1} \equiv \frac{\Psi'_{t+1}}{R_{t+1}} (c_{o,t+1} + M_{t+1}) > 0$ and $R_{t+1} \equiv 1 - \delta + f'_{t+1}(k_{t+1})$; and (ii) an optimal intergenerational allocation of resources $\frac{u'(c_{y,t})}{U'(c_{o,t})} = \phi_t^{-1} (1 + n_t)$ where $\phi_t \equiv \Phi_t / \Phi_{t-1}$ is the planner's relative concern of generation t with respect to generation $t - 1$.

Proof. See Appendix E.5. □

The following results are about

Proposition C.2. *Let $v_t \geq 0$ be the Lagrange multiplier for (24). A government provided with lump-sum transfers and uniform taxes on health, the optimal policy is characterized by (i) a tax on health given by*

$$\tau_t^h = \underbrace{\left(1 + \frac{U''_{t+1}}{U'_{t+1}} R_{t+1} v_t\right) \frac{\chi_{t+1}}{R_{t+1}}}_{\text{Externality}} + \underbrace{v_t \left(-\frac{u''_t}{u'_t} - |M'_{t+1}| \frac{U''_{t+1}}{U'_{t+1}}\right)}_{\text{Capital subsidy}} - \underbrace{v_t \frac{\Psi'_{t+1}}{\Psi_{t+1}}}_{\text{Completing markets}} \leq 0 \quad (\text{C.3})$$

and (ii) an optimal intergenerational allocation:

$$\phi_t^{-1}(1 + n_t) = (1 + \omega_{ig,t}) \cdot \frac{u'(c_{y,t})}{U'(c_{o,t})} \quad \text{where } \omega_{ig,t} \equiv \frac{-\frac{u''_t}{u'_t} v_t - \frac{U''_t}{U'_t} R_t v_{t-1}}{1 + \frac{U''_t}{U'_t} R_t v_{t-1}} > 0. \quad (\text{C.4})$$

Proof. See Appendix E.9. □

C.2 Linearized Systems and Stability Properties

The social planner's allocation solves: Given k_0 , for all $t \geq 0$

$$u'(c_{y,t}) = \phi_t^{-1}(1 + n_t) \cdot U'(c_{o,t}) \quad (\text{C.5})$$

$$u'(c_{y,t}) = [1 - \delta + f'_{t+1}(k_{t+1})] \cdot U'(c_{o,t+1}) \quad (\text{C.6})$$

$$u'(c_{y,t}) = \Psi'_{t+1}(h_t) (U(c_{o,t+1}) - c_{o,t+1} U'(c_{o,t+1})) - G'_{t+1}(h_t) U'(c_{o,t+1}). \quad (\text{C.7})$$

$$f_t(k_t) + (1 - \delta)k_t = c_{y,t} + h_t + \frac{\Psi_t(h_{t-1})c_{o,t} + G_t(h_{t-1})}{1 + n_t} + (1 + n_{t+1})k_{t+1} \quad (\text{C.8})$$

plus a proper transversality condition, where $G'_{t+1}(h) = \Psi'_{t+1}(h)M_{t+1}(h) + \Psi_{t+1}(h)M'_{t+1}(h)$. To simplify notation and connect our analysis to Assumption 4, I define $\Theta(c) \equiv \frac{U(c)}{U'(c)}$. Moreover, let $\Sigma \equiv -\frac{U''}{U'}c$ and $\sigma \equiv -\frac{u''}{u'}$.

Linear System. I now study the effects of a small perturbation to this system around the steady state. To capture a general form for shifts in the survival rate, I denote as ψ_t a general parameter governing Ψ_t (or M_t). Taking a first-order derivative around the steady state yields

$$\begin{aligned} u'' \cdot \partial c_{y,t} &= \phi^{-1}(1 + n)U'' \cdot \partial c_{o,t} + \phi^{-1}U' \cdot \partial n_t \\ u'' \cdot \partial c_{y,t} &= (1 - \delta + f')U'' \cdot \partial c_{o,t+1} + f''U' \cdot \partial k_{t+1} \\ u'' \cdot \partial c_{y,t} &= -(\Psi'c_o + G')U'' \cdot \partial c_{o,t+1} + (\Psi''c_o(\Theta - 1) - G'')U' \cdot \partial h_t \\ &\quad + \left(\frac{\partial \Psi'}{\partial \psi}c_o(\Theta - 1) - \frac{\partial G'}{\partial \psi}\right)U' \cdot \partial \psi_{t+1} \end{aligned}$$

$$(1 - \delta + f') \cdot \partial k_t = \partial c_{y,t} + \partial h_t + \frac{1}{1+n} ((\Psi' c_o + G') \cdot \partial h_{t-1} + \Psi \cdot \partial c_{o,t}) + (1+n) \partial k_{t+1} \\ - \frac{\Psi c_o + G}{1+n} \frac{1}{1+n} \cdot \partial n_t + k \cdot \partial n_{t+1} + \frac{1}{1+n} \left(\frac{\partial \Psi}{\partial \psi} c_o + \frac{\partial G}{\partial \psi} \right) \cdot \partial \psi_t$$

where I assume that only demographic variables, n_t and ψ_t , move. Let $\hat{x}_t \equiv \frac{\partial x_t}{x}$ be the percentage deviation from the approximated point (i.e., steady state).²³ Rearranging yields

$$\sigma \cdot \hat{c}_{y,t} = \Sigma \cdot \hat{c}_{o,t} - \hat{n}_t \\ \sigma \cdot \hat{c}_{y,t} = \Sigma \cdot \hat{c}_{o,t+1} + \omega_k \cdot \hat{k}_{t+1} \\ \sigma \cdot \hat{c}_{y,t} = -\omega_c \Sigma \cdot \hat{c}_{o,t+1} + \omega_h \cdot \hat{h}_t - \omega_\psi \cdot \partial \psi_{t+1} \\ \phi^{-1}(1+n) \cdot \hat{k}_t = \frac{c^y}{k} \cdot \hat{c}_{y,t} + \frac{\text{abs}_c}{k} \cdot \hat{c}_{o,t} + \frac{h}{k} \cdot \hat{h}_t + \phi^{-1} \omega_c \frac{h}{k} \cdot \hat{h}_{t-1} + (1+n) \cdot \hat{k}_{t+1} \\ - \frac{\text{abs}}{k} \cdot \hat{n}_t + (1+n) \cdot \hat{n}_{t+1} + \frac{\text{abs}_\psi}{k} \cdot \partial \psi_t$$

where $\omega_k \equiv -f'' k \frac{\phi}{1+n} > 0$, $\omega_c \equiv \frac{\Psi' c_o + G'}{\Psi' c_o (\Theta - 1) - G'} > 0$, $\omega_h \equiv -\frac{\Psi'' c_o (\Theta - 1) - G''}{\Psi' c_o (\Theta - 1) - G'} h > 0$, $\omega_\psi \equiv \frac{\frac{\partial \Psi'}{\partial \psi} c_o (\Theta - 1) - \frac{\partial G'}{\partial \psi}}{\Psi' c_o (\Theta - 1) - G'}$, $\text{abs} \equiv \frac{\Psi c_o + G}{1+n} > 0$ is the total demand from old individuals, $\text{abs}_c \equiv \frac{\Psi c_o}{1+n} > 0$ denotes the demand for consumption from the elderly, and $\text{abs}_\psi \equiv \frac{1}{1+n} \left(\frac{\partial \Psi}{\partial \psi} c_o + \frac{\partial G}{\partial \psi} \right)$ is the change in the aggregate demand due to changes in demographic consumption (or long-term care technology).

Difference Equation System. Combining the between- and within-generation intertemporal equations yields an Euler-type equation: For all $t \geq 0$,

$$\hat{c}_{o,t} = \hat{c}_{o,t+1} + \Sigma^{-1} \omega_k \cdot \hat{k}_{t+1} + \Sigma^{-1} \cdot \hat{n}_t. \quad (\text{C.9})$$

Combining the optimal health decision and the intergenerational optimality condition leads to

$$\hat{h}_t = \omega_h^{-1} \Sigma \cdot \hat{c}_{o,t} + \omega_h^{-1} \omega_c \Sigma \cdot \hat{c}_{o,t+1} + \omega_h^{-1} \cdot (\omega_\psi \cdot \partial \psi_{t+1} - \hat{n}_t).$$

Substituting the Euler-type equation into this equation yields

$$\hat{h}_t = \Sigma \frac{1 + \omega_c}{\omega_h} \cdot \hat{c}_{o,t} - \frac{\omega_c \omega_k}{\omega_h} \cdot \hat{k}_{t+1} + \frac{1}{\omega_h} \cdot (\omega_\psi \cdot \partial \psi_{t+1} - (1 + \omega_c) \cdot \hat{n}_t)$$

for all $t \geq 0$; and/or

$$\hat{h}_{t-1} = \Sigma \frac{1 + \omega_c}{\omega_h} \cdot \hat{c}_{o,t} + \frac{\omega_k}{\omega_h} \cdot \hat{k}_t + \frac{\omega_\psi}{\omega_h} \cdot \partial \psi_t$$

for all $t \geq 1$. Substituting the optimal intergenerational allocation and the last two equations into the resource constraint leads to: For $t = 0$,

$$\phi^{-1}(1+n) \cdot \hat{k}_0 = \phi^{-1} \omega_c \frac{h}{k} \cdot \hat{h}_{-1} + \underline{\varrho} \cdot \hat{c}_{o,0} + (1+n-\zeta) \cdot \hat{k}_1 + \hat{x}_0 \quad (\text{C.10})$$

²³Except for the fertility rate which is $\hat{n}_t = \frac{\partial n_t}{1+n}$.

where $\underline{\varrho} \equiv k^{-1} \left(\Sigma \left(\sigma^{-1} c_y + h \frac{1+\omega_c}{\omega_h} \right) + \text{abs}_c \right) > 0$, $\varsigma \equiv \frac{h}{k} \frac{\omega_c \omega_k}{\omega_h} > 0$, and

$$\hat{x}_0 \equiv - \left(\frac{c_y}{k} \frac{1}{\sigma} + \frac{h}{k} \frac{1+\omega_c}{\omega_h} + \frac{\text{abs}}{k} \right) \cdot \hat{n}_0 + (1+n) \cdot \hat{n}_1 + \frac{\text{abs}_\psi}{k} \cdot \partial \psi_0 + \frac{h}{k} \frac{\omega_\psi}{\omega_h} \cdot \partial \psi_1;$$

and, for all $t \geq 1$,

$$\phi^{-1} (1+n-\varsigma) \cdot \hat{k}_t = \varrho \cdot \hat{c}_{o,t} + (1+n-\varsigma) \cdot \hat{k}_{t+1} + \hat{x}_t \quad (\text{C.11})$$

where $\underline{\varrho} \equiv \varrho + (1+\omega_c) \frac{\Sigma}{\phi} \frac{h}{k} \frac{\omega_c}{\omega_h} > 0$ and

$$\hat{x}_t \equiv - \left(\frac{c_y}{k} \frac{1}{\sigma} + \frac{h}{k} \frac{1+\omega_c}{\omega_h} + \frac{\text{abs}}{k} \right) \cdot \hat{n}_t + (1+n) \cdot \hat{n}_{t+1} + \left(\frac{\text{abs}_\psi}{k} + \frac{1}{\phi} \frac{h}{k} \frac{\omega_c \omega_\psi}{\omega_h} \right) \cdot \partial \psi_t + \frac{h}{k} \frac{\omega_\psi}{\omega_h} \cdot \partial \psi_{t+1}.$$

The approximated solution is pinned down by (C.9), (C.10), and (C.11) together with initial condition \hat{k}_0 and \hat{h}_{-1} and a terminal condition $\lim_{t \rightarrow \infty} \hat{k}_{t+1} = 0$.

Solution and Stability Properties. Combining (C.9) and (C.11) for all $t \geq 1$, we obtain the following second-order difference equation

$$\begin{aligned} \frac{\Sigma^{-1} \varrho \cdot \hat{n}_t - \Delta \hat{x}_{t+1}}{1+n-\varsigma} &= \hat{k}_{t+2} - \left(\frac{\Sigma^{-1} \omega_k \varrho}{1+n-\varsigma} + 1 + \phi^{-1} \right) \cdot \hat{k}_{t+1} + \phi^{-1} \cdot \hat{k}_t \\ &= \left(L^{-2} - \left(\frac{\Sigma^{-1} \omega_k \varrho}{1+n-\varsigma} + 1 + \phi^{-1} \right) L^{-1} + \phi^{-1} \right) \cdot \hat{k}_t \\ &= (L^{-1} - \underline{\theta})(L^{-1} - \bar{\theta}) \cdot \hat{k}_t \\ &= (1 - \underline{\theta}L)(1 - \bar{\theta}L) \cdot \hat{k}_{t+1} \end{aligned}$$

where L and Δ denote the lag and difference operator, respectively, and $\underline{\theta}$ and $\bar{\theta}$ are the roots of the quadratic equation

$$0 = \mathcal{P}(k) \equiv k^2 - \left(\frac{\Sigma^{-1} \omega_k \varrho}{1+n-\varsigma} + 1 + \phi^{-1} \right) k + \phi^{-1}.$$

Note, moreover, that I have assumed that $1+n \neq \varsigma$. The quadratic polynomial $\mathcal{P}(k)$ has the following properties: (i) $\mathcal{P}(0) > 0$, (ii) $\mathcal{P}(1) = -\frac{\Sigma^{-1} \omega_k \varrho}{1+n-\varsigma}$, and (iii) $\mathcal{P}'(1) = 1 - \phi^{-1} - \frac{\Sigma^{-1} \omega_k \varrho}{1+n-\varsigma}$. Saddle-path stability for this system requires $1+n > \varsigma$, leading to $\mathcal{P}(1) < 0$ and guaranteeing $\underline{\theta} \in (0, 1)$ and $\bar{\theta} > 1$. Instead, if $1+n < \varsigma$, $\mathcal{P}(1) > 0$ with

$$\mathcal{P}'(1) = 1 + \frac{\Sigma^{-1} \omega_k \varrho}{\varsigma - (1+n)} + \phi^{-1} \left(\frac{1+\omega_c}{1-\frac{1+n}{\varsigma}} - 1 \right) > 0.$$

This means that when our saddle-path condition does not apply, $1+n < \varsigma$, both roots are lower than and any level of \hat{c}_0^0 will be consistent with the transversality condition. Throughout the main document, I impose the following assumption.

Assumption 5. *Around the steady state of the planner's allocation, $1+n > \varsigma$ where ς measures the rise in health stock h_t when k_{t+1} rises with a given $c_{o,t}$.*

It turns out that the planner's allocation under Assumption 5 is locally saddle-path stable. Assuming $1 + n > \varsigma$, the solution to the previous second-order difference equation is: For all $t \geq 1$

$$\widehat{k}_{t+1} = \underline{\theta} \cdot \widehat{k}_t - \bar{\theta}^{-1} \sum_{k=0}^{\infty} \bar{\theta}^{-k} X_{t+1+k} \quad (\text{C.12})$$

where $X_t \equiv \frac{\Sigma^{-1} \underline{\varrho} \cdot \widehat{n}_t - \Delta \widehat{x}_{t+1}}{1+n-\varsigma}$. Substituting this solution into the resource constraint at any $t \geq 1$,

$$\widehat{c}_{o,t} = \frac{(1-\phi\underline{\theta})(1+n-\varsigma)}{\phi\underline{\varrho}} \cdot \widehat{k}_t + \frac{1+n-\varsigma}{\bar{\theta}\underline{\varrho}} \sum_{k=0}^{\infty} \bar{\theta}^{-k} X_{t+1+k} - \frac{1}{\underline{\varrho}} \cdot \widehat{x}_t. \quad (\text{C.13})$$

Substituting this equation into the Euler equation for any $t \geq 1$ yields

$$\widehat{c}_{o,t-1} = \left(\Sigma^{-1} \omega_k + \frac{(1-\phi\underline{\theta})(1+n-\varsigma)}{\phi\underline{\varrho}} \right) \cdot \widehat{k}_t + \frac{1+n-\varsigma}{\bar{\theta}\underline{\varrho}} \sum_{k=0}^{\infty} \bar{\theta}^{-k} X_{t+1+k} - \frac{1}{\underline{\varrho}} \cdot \widehat{x}_t + \Sigma^{-1} \cdot \widehat{n}_t.$$

Evaluate the previous equation at $t = 1$ and replace it into the resource constraint at time 0:

$$\widehat{k}_1 = \frac{\phi^{-1} \left[(1+n) \cdot \widehat{k}_0 - \omega_c \frac{h}{k} \cdot \widehat{h}_{-1} \right] - \frac{1+n-\varsigma}{\bar{\theta}} \frac{\underline{\varrho}}{\underline{\varrho}} \sum_{k=0}^{\infty} \bar{\theta}^{-k} X_{2+k} + \frac{\underline{\varrho}}{\underline{\varrho}} \cdot \widehat{x}_1 - \widehat{x}_0 - \Sigma^{-1} \underline{\varrho} \cdot \widehat{n}_1}{\Sigma^{-1} \underline{\varrho} \omega_k + (1+n-\varsigma) \left(1 + \phi^{-1} (1-\phi\underline{\theta}) \frac{\underline{\varrho}}{\underline{\varrho}} \right)}. \quad (\text{C.14})$$

Moreover, recall that the optimal health investments are given by

$$\widehat{h}_t = \frac{\Sigma}{\omega_h} \cdot \widehat{c}_{o,t} + \frac{\omega_c \Sigma}{\omega_h} \cdot \widehat{c}_{o,t+1} + \frac{1}{\omega_h} \cdot (\omega_\psi \cdot \partial \psi_{t+1} - \widehat{n}_t). \quad (\text{C.15})$$

C.3 Phase Diagram for Social Planner's Allocation

Considering the assumptions in Proposition 6, the optimal planner's allocation solves the dynamic system given by equations (C.5)-(C.8) plus the appropriate transversality condition. Let $\mathcal{C}_{y,t}(c) \equiv (u')^{-1} \left(\phi_t^{-1} (1+n_t) U'(c) \right)$ and $\mathcal{H}_t(c_t^o, c_{t+1}^o)$ is the level of h satisfying (C.7) for some given levels of $c_{o,t}$ and $c_{o,t+1}$ (with $c_{y,t} = \mathcal{C}_{y,t}(c_{o,t})$). Then, the two-difference-equation system becomes

$$\frac{U'(c_{o,t})}{U'(c_{o,t+1})} = \frac{\phi_t}{1+n_t} [1 - \delta + f'(k_{t+1})] \quad (\text{C.16})$$

$$(1+n_{t+1})(k_{t+1} - k_t) = f(k_t) - (n_{t+1} + \delta)k_t - \mathcal{C}_{y,t}(c_{o,t}) - \mathcal{H}_t(c_{o,t}, c_{o,t+1}) - \frac{1}{1+n_t} [\Psi_t(\mathcal{H}_{t-1}(c_{o,t-1}, c_{o,t})) c_{o,t} + G_t(\mathcal{H}_{t-1}(c_{o,t-1}, c_{o,t}))]. \quad (\text{C.17})$$

Note that $\mathcal{H}(x, y)$ is increasing in both arguments. In fact,

$$\mathcal{H}_{t,1}(c_{o,t}, c_{o,t+1}) = \frac{\Sigma}{\omega_h} \frac{h_t}{c_{o,t}} > 0 \quad \text{and} \quad \mathcal{H}_{t,2}(c_{o,t}, c_{o,t+1}) = \frac{\omega_c \Sigma}{\omega_h} \frac{h_t}{c_{o,t+1}} > 0.$$

In this model, we can construct a "phase diagram" in the $(k_t, c_{o,t})$ -space as follows: Suppose all exogenous variables are constant and compute the isoclines.

Fixed Consumption Path. When $c_{o,t} = c_{o,t+1}$, k_{t+1} is independent of $c_{o,t}$, say k , and pinned down by (C.16). Then, the combination of k_t and $c_{o,t}$ that keeps consumption constant obeys an increasing relationship given by

$$f(k_t) + (1 - \delta)k_t = C_y(c_{o,t}) + \mathcal{H}(c_{o,t}, c_{o,t}) \\ + \frac{1}{1+n} [\Psi(\mathcal{H}(c_{o,t}, c_{o,t})) c_{o,t} + G(\mathcal{H}(c_{o,t}, c_{o,t}))] + (1+n)k.$$

The slope of this locus is given by

$$\left. \frac{\partial c_{o,t}}{\partial k_t} \right|_{c_{o,t}=c_{o,t+1}} = \frac{1 - \delta + f'}{\frac{\Sigma c_y}{\sigma c_o} + \left(1 + \frac{\Psi' c_o + G'}{1+n}\right) (\mathcal{H}_1 + \mathcal{H}_2)} > 0.$$

Moreover, as $c_{o,t} \rightarrow 0$, the stock of capital approximates to $f(k_t) + (1 - \delta)k_t = \frac{G(0)}{1+n} + (1+n)k$.

Fixed Capital Path. When $k_t = k_{t+1}$, $c_{o,t}$ and $c_{o,t+1}$ respect an increasing relationship with a slope that depends on k_{t+1} . Specifically, from (C.16), we can compute the following functions: $c_{o,t+1} = q_+(c_{o,t}, k_{t+1})$ and $c_{o,t} = q_-(c_{o,t+1}, k_{t+1})$. As a result, the locus $(c_{o,t}, k_t)$ keeping the stock of capital constant satisfies

$$f(k_t) - (n + \delta)k_t = C_y(c_{o,t}) + \mathcal{H}(c_{o,t}, q_+(c_{o,t}, k_t)) \\ + \frac{1}{1+n} [\Psi(\mathcal{H}(q_-(c_{o,t}, k_t), c_{o,t})) c_{o,t} + G(\mathcal{H}(q_-(c_{o,t}, k_t), c_{o,t}))].$$

The slope of this locus is

$$\left. \frac{\partial c_{o,t}}{\partial k_t} \right|_{k_t=k_{t+1}} = \frac{f' - (n + \delta) - \mathcal{H}_2 q_{+,2} - \frac{\Psi' c_o + G'}{1+n} \mathcal{H}_1 q_{-,2}}{\frac{\Sigma c_y}{\sigma c_o} + \mathcal{H}_1 + \mathcal{H}_2 q_{+,1} + \frac{\Psi' c_o + G'}{1+n} (\mathcal{H}_1 q_{-,1} + \mathcal{H}_2)} \geq 0$$

where $q_{+,1} = \frac{\Sigma_t}{\Sigma_{t+1}} > 0$, $q_{-,1} = q_{+,1}^{-1}$, $q_{+,2} = \frac{\phi}{1+n} \frac{U'_{t+1}}{U'_t} \frac{c_{o,t+1}}{\Sigma_{t+1}} f''_{t+1} < 0$, and $q_{-,2} = -\frac{\phi}{1+n} \frac{U'_{t+1}}{U'_t} \frac{c_{o,t}}{\Sigma_t} f''_{t+1} > 0$. Evaluating at the steady state yields $q_{-,1} = q_{+,1} = 1$, $q_{-,2} = -q_{+,2} = -\frac{\phi}{1+n} \frac{c_o}{\Sigma} f''$ and

$$f' - (n + \delta) - \mathcal{H}_2 q_{+,2} - \frac{\Psi' c_o + G'}{1+n} \mathcal{H}_1 q_{-,2} = \frac{1 - \phi}{\phi} (1 + n - \mathcal{H}_2 q_{-,2}).$$

Finally, as $c_{o,t} \rightarrow 0$, the stock of capital approximates to $f(k_t) - (n + \delta)k_t = \frac{G(0)}{1+n}$.

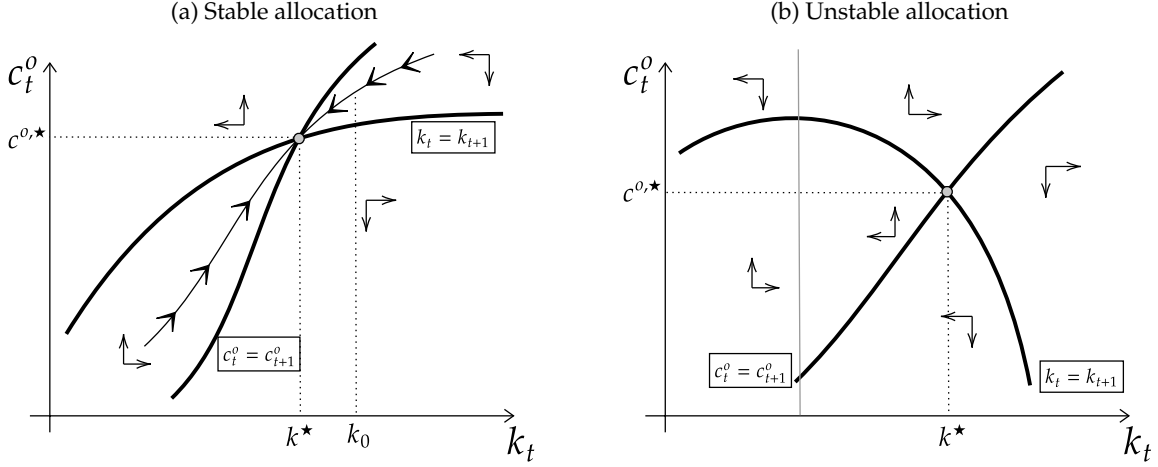
Stable Solution. To find the stability properties of this system, I linearize the dynamic system around a steady state as in Appendix C.2. Therefore, around the steady state under Assumption 5:

$$\left. \frac{\partial c_{o,t}}{\partial k_t} \right|_{k_t=k_{t+1}} < \left. \frac{\partial c_{o,t}}{\partial k_t} \right|_{c_{o,t}=c_{o,t+1}}.$$

Phase Diagram. Under general parameterization (including the ones in Assumption 3), we can use the “heuristics” of a phase diagram applied to my model. In particular, Figure C.1 shows this phase diagram and, unsurprisingly, this looks similar to the traditional neoclassical model’s phase diagram.

Comparative Dynamics. The dynamics of the planner’s allocation after any type shock can be explored as in the usual way. Figure C.2 shows the dynamics of the planner’s allocation under a shift in the survival rate that raises both the level of survival and the productivity of health investments.

Figure C.1: Phase diagram for the social planner's allocation



Note. The figure illustrates the dynamic adjustment of capital and old-age consumption under the social planner's allocation. The intersection of the $c_{o,t} = c_{o,t+1}$ and $k_t = k_{t+1}$ loci defines the steady state. Arrows indicate the direction of motion implied by the difference equation system. Panel (a) shows a stable allocation, while panel (b) shows an unstable one.

C.4 Numerical Solution

Although the below-described algorithms work under more general calibrations, the numerical illustrations in Section 4 assumes Assumption 3.

Stationary Allocation. Under Assumption 3, the stationary allocation for the planner is given by

$$\omega = \phi^{-1}(1 + n) \quad (\text{C.18})$$

$$\omega = 1 - \delta + f'(k) \quad (\text{C.19})$$

$$\omega = \Psi'(h)c_o[\Theta(c_o) - 1] - G'(h) \quad (\text{C.20})$$

$$f(k) - (n + \delta)k = c_y + h + \frac{\Psi(h)c_o + G(h)}{1 + n} \quad (\text{C.21})$$

where $\omega \equiv \frac{u'(c_y)}{U'(c_o)}$ is the ratios of marginal utilities.

Pareto frontier. Combining equations (C.19) and (C.21) yields the Pareto frontier (for a given h) for consumptions (c_y, c_o) as follows: Fix $\omega > (1 - \delta)^{1/\sigma}$ and compute c_o from

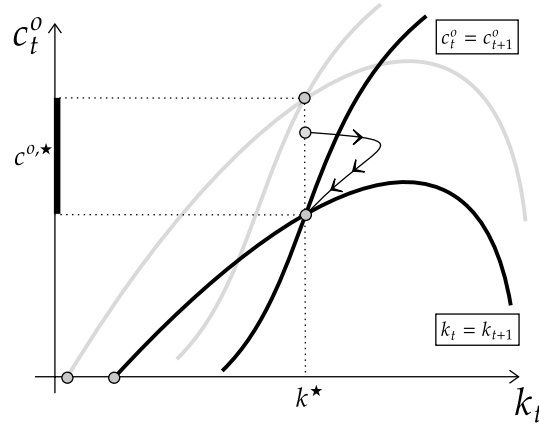
$$\mathcal{C}(\omega, c_o) + h + \frac{\Psi(h)c_o + G(h)}{1 + n} = f(\kappa(\omega)) - (n + \delta)\kappa(\omega) \quad (\text{C.22})$$

where $\kappa(\omega) \equiv [f']^{-1}(\omega - 1 + \delta)$ and $\mathcal{C}(\omega, c_o) \equiv [u']^{-1}(U'(c_o) \cdot \omega)$.

Allocation. From the system of non-linear equations, ω is given so is $k = \kappa(\omega)$. Thus, the consumption of old individuals and the health stock (c_o, h) together solves (C.20) and (C.22).

Consider now the following constrained allocations. First, "Constrained I", suppose that the health stock is given at h . Since this means that (C.20) is not part of the solution, c_o is given by (C.22). Second,

Figure C.2: Comparative dynamics for an increment in survival rate



Note. This figure illustrates the effects of an increase in the survival rate using the phase diagram. Light curves correspond to the initial steady state, while dark curves depict the loci after an increase in survival. An increase in survival shifts the loci and induces transitional dynamics characterized by an initial adjustment in capital and old-age consumption, followed by convergence to the new steady state along the stable path. Arrows indicate the direction of motion.

“Constrained II”, suppose that the government does not consider the demographic externality, $\chi_{h,t+1} = 0$. Then, (c_o, h) together solves (C.22) and

$$\omega = \Psi'(h)c_o\Theta(c_o) - \Psi(h)M'(h).$$

Special case. For the sake of illustrating, take Assumption 3 under the following calibration $\Psi(h) = \psi h^{\xi h}$, $n = 0$, $\delta = 1$, $Z = 1$, $\sigma < 1$, $\bar{U} = 0$, $\xi_m = \xi_h = \xi$, and $\phi = 1$. Under this calibration $\omega_c = 1$, $k = (\alpha\phi)^{\frac{1}{1-\alpha}}$, and the unconstrained allocation (c_o, h) solves

$$c_o = \frac{1}{1 + \psi h^{\xi}} [k^{\alpha} - k - \psi \bar{M} - h] \text{ and } c_o = \frac{1 - \sigma}{\sigma \xi \psi} h^{1-\xi}.$$

While the “Constrained II” allocation remove the second equation and consider $1 = \frac{\xi \bar{\Psi}}{1-\sigma} h^{\xi-1} c_o + \xi \bar{\Psi} \bar{M} h^{-1}$ instead.

Dynamic Allocation. Given $k_0 > 0$, and $h_{-1} > 0$, the optimal planner’s allocation solves

$$\begin{aligned} u'(c_{y,t})/U'(c_{o,t}) &= \phi_t^{-1}(1 + n_t) \\ u'(c_{y,t})/U'(c_{o,t+1}) &= 1 - \delta + f'_{t+1}(k_{t+1}) \\ u'(c_{y,t})/U'(c_{o,t+1}) &= \Psi'_{t+1}(h_t)c_{o,t+1}[\Theta(c_{o,t+1}) - 1] - G'_{t+1}(h_t) \\ f_t(k_t) + (1 - \delta)k_t &= c_{y,t} + h_t + \frac{\Psi_t(h_{t-1})c_t^o + G_t(h_{t-1})}{1 + n_t} + (1 + n_{t+1})k_{t+1} \end{aligned}$$

with a proper transversality condition. This system can be solved numerically as follows:

1. Guess an initial consumption for olds $c_{o,0} > 0$.
2. For all $t = 0, 1, \dots, T$, given $(c_{o,t}, k_t, h_{t-1})$, solve for $(c_{o,t+1}, k_{t+1}, h_t)$ as follows

(a) Find the consumption for young:

$$c_{y,t} = [u']^{-1} \left((1 + n_t) U'(c_{o,t}) / \phi_t \right).$$

(b) Compute $\text{aux}_{0,t} = c_{y,t} + \frac{\Psi_t(h_{t-1})c_{o,t}^0 + G_t(h_{t-1})}{1+n_t}$ and $\text{aux}_{1,t} = f_t(k_t) + (1 - \delta)k_t$.

(c) Define $c(k) = [U']^{-1} \left(\frac{u'(c_{y,t})}{1 - \delta + f'_{t+1}(k)} \right)$.

(d) Find (k_{t+1}, h_t) by solving the two-nonlinear equation system

$$\begin{aligned} \text{aux}_{1,t} &= \text{aux}_{0,t} + h_t + (1 + n_{t+1})k_{t+1} \\ \frac{u'(c_{y,t})}{U'(c(k_{t+1}))} &= \Psi'_{t+1}(h_t) \cdot c(k_{t+1}) [\Theta(c(k_{t+1})) - 1] - G'_{t+1}(h_t). \end{aligned}$$

3. Check if $|k_{T+1} - k| \simeq 0$. If not, adjust the guess $c_{o,0}$.

Generally, K_0 is an arbitrary level so is k_0 . However, whenever I assume that the economy is in a steady state prior to the exogenous shifts k_0 is a function of the previous steady state of capital, k_- . Let $N_{y,0}^e$ and $k_0^e = k_-$ be the expected population size and capital stock per labor before the unanticipated movements; then, the initial aggregate stock of capital is $K_0 = k_- N_{y,0}^e$. As a result, $k_0 = k_- \frac{1+n_-}{1+n_0} > 0$.

D Omitted Derivations for Section 5

D.1 Distribution and Competitive Equilibrium

Distributions. The distribution over idiosyncratic states $\mu_{j,t}(\mathbf{x})$ is a crucial object in describing the equilibrium in this economy. The law of motion of this distribution is as follows: Given $\mu_{j,t-1}(\cdot)$ and the initial level of assets at t , $a_{t,0}$, the distribution $\mu_{j,t}(\cdot)$ for workers is

$$\mu_{j,t}(a_+, z_p, z_{s,+}) = \begin{cases} \frac{\iota_t}{1+n_t} \cdot \mathbb{P}_{p,t}(z_p) \mathbb{P}_s(z_s) \cdot \mathbb{I}_{a=a_{t,0}} & j = 0 \\ \frac{\psi_{j-1,t-1}(z_p)}{1+n_t} \cdot \int_a \int_{z_s} \mu_{j-1,t-1}(\mathbf{x}) \mathbb{P}(z_{s,+} | z_s) \mathbb{I}_{a_{j-1,t-1}^+(\mathbf{x})=a^+} \partial z_s \partial a & j \in \mathcal{J}^w / \{0\} \end{cases} \quad (\text{D.1})$$

while, for retirees,

$$\mu_{j,t}(a_+, z_p) = \begin{cases} \frac{\psi_{j-1,t-1}(z_p)}{1+n_t} \cdot \int_a \int_{z_s} \mu_{j-1,t-1}(\mathbf{x}) \mathbb{I}_{a_{j-1,t-1}^+(\mathbf{x})=a^+} \partial z_s \partial a & j = J^r \\ \frac{\psi_{j-1,t-1}(z_p)}{1+n_t} \cdot \int_a \mu_{j-1,t-1}(\mathbf{x}) \mathbb{I}_{a_{j-1,t-1}^+(\mathbf{x})=a^+} \partial a & j = \mathcal{J}^r / \{J^r\} \end{cases}. \quad (\text{D.2})$$

Demographics. Demography is determined by the exogenous process $\{\iota_t, \{\psi_{j,t}\}_{j=0}^J\}_{t \geq 0}$ and a given initial demographic structure. For all $j \in \mathcal{J}$, let $\bar{\mu}_{j,t}(z_p)$ be the fraction of z_p - individuals aged j at time t . According to (D.1) and (D.2), the age distribution satisfies the following system of

equations: $\bar{\mu}_{0,t} = \frac{\iota_t}{1+n_t} \mathbb{P}_{p,t}(z_p)$ and $\bar{\mu}_{j,t}(z_p) = \frac{\psi_{j-1,t-1}(z_p)}{1+n_t} \bar{\mu}_{j-1,t-1}(z_p)$ for all $j \in \mathcal{J}/\{0\}$. Moreover, any demographic path $\{n_t, \{\bar{\mu}_{j,t}(z_p)\}_{j=0}^J\}_{t \geq 0}$ should verify the following equations:

$$n_t = \iota_t - \sum_{j=0}^J \int_{z_p} [1 - \psi_{j,t-1}(z_p)] \bar{\mu}_{j,t-1}(z_p) \partial z_p. \quad (\text{D.3})$$

The dependency ratio is the ratio of the number of retirees over the number of workers, i.e.,

$$d_t \equiv \frac{\sum_{j \in \mathcal{J}^r} \int_{z_p, t-j} \bar{\mu}_{j,t}(z_p, t-j) \partial z_p, t-j}{\sum_{j \in \mathcal{J}^w} \int_{z_p, t-j} \bar{\mu}_{j,t}(z_p, t-j) \partial z_p, t-j}. \quad (\text{D.4})$$

Competitive Equilibrium. In my model, there are two types of exogenous variations. First, the economy experiences (structural) changes in demographic variables $\{\iota_t, \{\psi_{j,t}(\cdot)\}_{j=0}^J\}$, permanent income inequality \mathbb{P}_t^p , technology progress Z_t , and/or financial conditions captured by borrowing limits $\{a_{j,t}\}_{j=0}^J$. Second, the government modifies the trajectory of any of the components in $\mathcal{G}_0 \equiv \{B_{g,t+1}, G_t, \tau_{k,t}, \mathcal{T}_{j,t}, \mathcal{S}_{j,t}\}_{t \geq 0}$, letting the lump-sum taxes τ_t vary so that the government's budget constraint holds.

Definition 4 (Competitive Equilibrium). Given the exogenous sequence $\{\mathbb{P}_t^p, \iota_t, \gamma_t, \{a_{j,t}, \psi_{j,t}\}_j\}$, policy instrument path $\{g_t, \tau_t, \tau_t^k, b_{t+1}^g, \{\mathcal{T}_{j,t}, \mathcal{S}_{j,t}\}_j\}$, and initial conditions $\{\mu_{-1}, b_{-1}, k_{-1}\}$, a competitive equilibrium is an object

$$\mathcal{E} \equiv \left\{ \left\{ V_{j,t}(\cdot), c_{j,t}(\cdot), a_{j,t}^+(\cdot), \mu_{j,t}(\cdot) \right\}_{j=0}^J, n_t, r_t, w_t, h_t, \tau_t, y_t, k_t, \ell_t \right\}_t$$

such that (i) For all j , the value functions $\{V_{j,t}\}_j$ satisfies (29) and $\{c_{j,t}, a_{j,t}^+\}_j$ are the corresponding policy functions, (ii) For all t , $\{\mu_{j,t}\}_j$, n_t , and b_t^g are consistent with equations (30), (D.1), (D.2), and (31), (iii) For all t , given r_t and w_t , firms maximize profits: $y_t = F(k_t/\ell_t, Z_t)\ell_t$, $r_t = F_K(k_t/\ell_t, Z_t) - \delta$, and $w_t = F(k_t/\ell_t, Z_t) - F_K(k_t/\ell_t, Z_t)k_t/\ell_t$, (iv) For all t ,

$$s_t + g_t + (1 + r_t)b_t^g = (1 + n_{t+1})b_{t+1}^g + \tau_t + \tau_t^k r_t k_t$$

where $s_t = \sum_{j \in \mathcal{J}^r} \int_{z_p} \mathcal{S}_{j,t}(z^p) \mu_{j,t}(z^p) \partial z^p$, $g_t = G_t/N_t$ and $b_t^g = B_t^g/N_t$, and (v) For all t , markets clear

$$\ell_t = \sum_{j \in \mathcal{J}^w} \mu_{j,t} \quad \text{and} \quad k_t + b_t = \int_a a \sum_{j \in \mathcal{J}} \mu_{j,t}(a) \partial a.$$

Normalized Equilibrium Conditions. Let $u(c)$ and $U(a)$ such that $u_t(c) = u(c/Z_t)$ and $U_t(a) = U(a/Z_t)$. Similarly, suppose $\mathcal{T}_{j,t}(\omega) \equiv Z_t \widehat{\mathcal{T}}_{j,t}(\omega/Z_t)$ and $\mathcal{S}_{j,t}(z) \equiv Z_t \widehat{\mathcal{S}}_{j,t}(z)$. Now, I describe the normalized model.

Normalized Household's Problem. For retirees, let $V_{j,t}(\mathbf{x}; \mathbf{X})$ denote the continuing discounted utility which is the solution to the following problem: For $j = J - 1, \dots, J'$,

$$V_{j,t}(\mathbf{x}; \mathbf{X}_t) = \max_{c, a_+ \geq \underline{a}_{j+1,t}} \left\{ u(c) + \beta \left[\psi_{j,t}(z_p) V_{j+1,t+1}(a_+, z^p; \mathbf{X}_{t+1}) + [1 - \psi_{j,t}(z_p)] U((1 + \gamma_{t+1})a_+) \right] \right\}$$

subject to

$$c + (1 + \gamma_{t+1})a_+ = (1 + \tilde{r}_t)a + b_{q,t} + \widehat{S}_{j,t}(z_p)$$

where $\widehat{a}_{j+1,t} = a_{j+1,t}/Z_t$ and the other variables should be interpreted in efficiency units (i.e., in terms relative to Z_t). For workers, $V_{j,t}(\mathbf{x}; \mathbf{X})$ is the backward solution to: For $j = J' - 1, \dots, 0$,

$$V_{j,t}(\mathbf{x}; \mathbf{X}_t) = \max_{c, a_+ \geq \widehat{a}_{j+1,t}} \left\{ u(c) + \beta \left[\psi_{j,t}(z_p) \mathbb{E}_{z_+^s | z^s} [V_{j+1,t+1}(a_+, z_p, z_{s,+}; \mathbf{X}_{t+1})] + [1 - \psi_{j,t}(z_p)] U((1 + \gamma_{t+1})a_+) \right] \right\}$$

subject to

$$c + (1 + \gamma_{t+1})a_+ = (1 + \tilde{r}_t)a + b_{q,t} + \widehat{\mathcal{Y}}_{j,t}(\widehat{w}_t \ell_j(z_p, z_s)).$$

Bequests. The level of inheritances received for each agent at t , $b_{q,t}$, is

$$b_{q,t} = \frac{1 + \tilde{r}_t}{(1 + \gamma_t)(1 + n_t)} \cdot \sum_{j=0}^J \int_{\mathbf{x}} [1 - \psi_{j,t-1}(z_p)] a_{j,t-1}^+(\mathbf{x}) \mu_{j,t-1}(\mathbf{x}) d\mathbf{x}.$$

Final-Good Producers. The normalized version of the production function and firms' FOC are:

$$\begin{aligned} y_t &= F(k_t, l_t) \\ r_t + \delta &= F_K(k_t, l_t) \\ \widehat{w}_t &= F_L(k_t, l_t) \end{aligned}$$

Government. The government budget constraint is

$$s_t + g_t + (1 + r_t)b_t = (1 + \gamma_{t+1})(1 + n_{t+1})b_{t+1} + \tau_{k,t} r_t k_t + t_t + \tau_t \frac{1}{1 + d_t}$$

where

$$\begin{aligned} s_t &= \sum_{j \in \mathcal{J}_r} \int_{z_{p,t-j}} \widehat{S}_{j,t}(z_{p,t-j}) \bar{\mu}_{j,t}(z_{p,t-j}) \partial z_{p,t-j} \\ t_t &= \sum_{j \in \mathcal{J}_w} \int_{z_{p,t-j}} \int_{z_s} \widehat{T}_{j,t}(\widehat{w}_t e_j(z_{p,t-j}, z_s)) \bar{\mu}_{j,t}(z_{p,t-j}, z_s) \partial z_{p,t-j} \partial z_s. \end{aligned}$$

D.2 Numerical Solution

D.2.1 Basic EGM

Recursive Problem. Recall that the recursive problem is

$$V_j(a; \mathbf{z}) = \max_{c, a_+ \geq \widehat{a}_{j+1}} \left\{ u_j(c) + \beta \cdot W_j(a_+; \mathbf{z}) \right\} \text{ s.t.: } \Gamma a_+ = R^{\text{net}} \cdot a + y_j(\mathbf{z}) - c.$$

The optimality conditions for the consumption-savings problem are

$$\Gamma \cdot \partial_c u_j(c) \geq \beta \cdot \partial_a W_j(a_+; \mathbf{z})$$

plus the evolution of assets (budget constraint) and the borrowing constraint. Let denote the policy functions as $c_j(a; z)$ and $a_{+,j}(a; z)$. The envelope condition is

$$\partial_a V_j(a; z) \equiv R^{\text{net}} \cdot \partial_c u_j(c_j(a; z)).$$

Algorithm. Note that $\psi_j(z) = 0$ for all z , we can compute this continuation values:

$$W_j(a; z) = U_{j+1}(\Gamma a) \quad \text{and} \quad \partial_a W_j(a; z) = \Gamma \cdot \partial_a U_{j+1}(\Gamma a).$$

Conceptually, we should be able to solve the problem by backward solution using a root-finding algorithm. However, we can avoid root finding methods by using Endogenous Grid Method (EGM). Then, the value function and policy functions are solved as follows:

1. Given $W_j(a; z)$ and $\partial_a W_j(a; z)$, we can use EGM to compute $c_j(a; z)$ and $a_{+,j}(a; z)$:

(a) For each z (omitted in the following notation), solve for c^{endo} in

$$\Gamma \cdot \partial_c u_j(c^{\text{endo}}) = \beta \cdot \partial_a W_j(a).$$

(b) Find the endogenous grid

$$a^{\text{endo}} = \frac{\Gamma \cdot a + c^{\text{endo}} - y_j}{R^{\text{net}}}.$$

(c) Check borrowing constraints. Let $\underline{a}^{\text{endo}} \equiv a^{\text{endo}}(\underline{a}_{j+1})$ be the level of assets that ‘‘rationalizes’’ hitting the borrowing constraint. Because of monotonicity in the asset policy function, for any value in the grid at j such that $a \leq \underline{a}^{\text{endo}}$ must have the following solution: $a_{+,j}(a) = \underline{a}_{j+1}$.

2. By the envelope condition, we can compute $\partial_a V_j(a; z)$ and, by definition,

$$V_j(a; z) = u_j(c_j(a; z)) + \beta \cdot W_j(a_{+,j}(a; z); z).$$

3. Next, we can compute

$$W_{j-1}(a; z) = \psi_{j-1}(z) \cdot \mathbb{E}_+ [V_j(a; z_+) | z] + [1 - \psi_{j-1}(z)] \cdot U_j(\Gamma a)$$

and

$$\partial_a W_{j-1}(a; z) = \psi_{j-1}(z) \cdot \mathbb{E}_+ [\partial_a V_j(a; z_+) | z] + [1 - \psi_{j-1}(z)] \cdot \Gamma \partial_a U_j(\Gamma a).$$

D.2.2 Endogenous Health Formation

Recursive Problem. Adding health investments, the idiosyncratic state becomes (a, h, z) and the continuation value

$$W_j(a, h; z) \equiv \psi_j(h, z) \cdot \mathbb{E}_+ [V_{j+1}(a, h; z_+) | z] + [1 - \psi_j(h, z)] \cdot U(a).$$

The optimization problem is

$$V_j(a, h; \mathbf{z}) = \max_{c, \iota, a_+ \geq \underline{a}_{j+1}} \{u_j(c) + \omega_j(h_+) + \beta \cdot W_j(a_+, h_+; \mathbf{z})\}$$

subject to

$$a_+ = R^{\text{net}} \cdot a + \tilde{y}_j(h; \mathbf{z}) - m_j(h; \mathbf{z}) - c - (1 + \tau_j^h) \cdot \iota \text{ and } h_+ = (1 - \delta_j^h) \cdot h + \mathcal{H}_j(\iota; \mathbf{z}).$$

The optimality conditions are

$$\begin{aligned} \partial_c u_j(c) &\geq \beta \cdot \partial_a W_j(a_+, h_+; \mathbf{z}) \\ (1 + \tau_j^h) \cdot \partial_c u_j(c) &= \partial_\iota \mathcal{H}_j(\iota; \mathbf{z}) \cdot [\partial_h \omega_j(h_+) + \beta \cdot \partial_h W_j(a_+, h_+; \mathbf{z})] \end{aligned}$$

plus the evolution of assets and health and the borrowing limit. Let denote the policy functions as $c_j(a, h; \mathbf{z})$, $a_{+,j}(a, h; \mathbf{z})$, and $h_{+,j}(a, h; \mathbf{z})$. Then, the envelope conditions are

$$\begin{aligned} \partial_a V_j(a, h; \mathbf{z}) &\equiv R^{\text{net}} \cdot \partial_c u_j(c_j(a, h; \mathbf{z})) \\ \partial_h V_j(a, h; \mathbf{z}) &\equiv [\partial_h \tilde{y}_j(h; \mathbf{z}) - \partial_h m_j(h; \mathbf{z})] \cdot \partial_c u_j(c_j(a, h; \mathbf{z})) \\ &+ (1 - \delta_j^h) \cdot [\partial_h \omega_j(h_{+,j}(a, h; \mathbf{z})) + \beta \cdot \partial_h W_j(a_{+,j}(a, h; \mathbf{z}), h_{+,j}(a, h; \mathbf{z}); \mathbf{z})] \end{aligned}$$

Based on our assumptions, for retirees $\tilde{y}_j(h; \mathbf{z}) = \mathcal{S}_j(\bar{y})$, while for workers $\tilde{y}_j(h; \mathbf{z}) = \mathcal{Y}_j(y_j(h; \mathbf{z}))$ where $y_j(h; \mathbf{z}) = \exp(\xi_z h) \epsilon_j z_p z_s$. Thus,

$$\partial_h \tilde{y}_j(h; \mathbf{z}) = \xi_z y_j(h; \mathbf{z}) \cdot \frac{\partial \mathcal{Y}_j(y_j(h; \mathbf{z}))}{\partial y}$$

Algorithm. I om it the dependence on \mathbf{z} for convenience. Then, given a and h , we should solve for c^{endo} and ι^{endo} in

$$\begin{aligned} \partial_c u_j(c^{\text{endo}}) &= \beta \cdot \partial_a W_j(a, h) \\ \partial_\iota \mathcal{H}_j(\iota^{\text{endo}}) &= \frac{\beta(1 + \tau_j^h) \cdot \partial_a W_j(a, h)}{\partial_h \omega_j(h) + \beta \cdot \partial_h W_j(a, h)}. \end{aligned}$$

Once we have computed these endogenous objects, we can compute the endogenous grids for the state variables: the endogenous grid for health stock is $h^{\text{endo}} = \frac{h_+ - \mathcal{H}_j(\iota^{\text{endo}})}{1 - \delta_j^h}$, while the grid for assets is

$$a^{\text{endo}} = \frac{a + c^{\text{endo}} + (1 + \tau_j^h) \cdot \iota^{\text{endo}} + m_j(h^{\text{endo}}) - y_j(h^{\text{endo}})}{R^{\text{net}}}.$$

Next, we should interpolate $(a^{\text{endo}}, h^{\text{endo}}) \rightarrow (a, h)$ over our defined grid (a, h) . If the interpolation at the query grid (a_+, h_+) is such that $a_+ \geq \underline{a}_{j+1}$, then we found a solution; otherwise, we should solve the following root-finding problem at grid (a, h) :

$$\tilde{c}(\iota) + (1 + \tau_j^h) \cdot \iota = (1 + \tilde{r}) \cdot a + \tilde{y}_j(h) - m_j(h) - \underline{a}_{j+1}$$

where $\tilde{c}(\iota)$ is the solution to

$$(1 + \tau_j^h) \cdot \partial_c u_j(c) = \partial_\iota \mathcal{H}_j(\iota) \cdot \left[\partial_h \omega_j(h + \mathcal{H}_j(\iota)) + \beta \cdot \partial_h W_j(\underline{a}_{j+1}, h + \mathcal{H}_j(\iota)) \right].$$

Having found the policy functions $c_j(a, h; \mathbf{z})$, $a_{+,j}(a, h; \mathbf{z})$, and $h_{+,j}(a, h; \mathbf{z})$, we can compute $\partial_a V_j$ and $\partial_h V_j$ via the envelope conditions and, by definition,

$$V_j(a, h; \mathbf{z}) = u_j(c_j(a, h; \mathbf{z})) + \omega_j(h_{+,j}(a, h; \mathbf{z})) + \beta \cdot W_j(a_{+,j}(a, h; \mathbf{z}), h_{+,j}(a, h; \mathbf{z}); \mathbf{z}).$$

Next, we can compute the continuation value

$$W_{j-1}(a, h; \mathbf{z}) = \psi_{j-1}(h, \mathbf{z}) \cdot \mathbb{E}[V_j(a, h; \mathbf{z}_+) | \mathbf{z}] + [1 - \psi_{j-1}(h, \mathbf{z})] \cdot U_j(a)$$

and the associated derivatives

$$\begin{aligned} \partial_a W_{j-1}(a, h; \mathbf{z}) &= \psi_{j-1}(h, \mathbf{z}) \cdot \mathbb{E}_+[\partial_a V_j(a, h; \mathbf{z}_+) | \mathbf{z}] + [1 - \psi_{j-1}(h, \mathbf{z})] \cdot \partial_a U_j(a) \\ \partial_h W_{j-1}(a, h; \mathbf{z}) &= \psi_{j-1}(h, \mathbf{z}) \cdot \mathbb{E}_+[\partial_h V_j(a, h; \mathbf{z}_+) | \mathbf{z}] + \partial_h \psi_{j-1}(h, \mathbf{z}) \cdot [\mathbb{E}_+[V_j(a, h; \mathbf{z}_+) | \mathbf{z}] - U_j(a)]. \end{aligned}$$

D.3 Welfare Measures

In this section, I discuss the welfare consequences of changes in τ^h . Let V_j and μ_j be the value function and distribution before the introduction of the policy reform, and V_j^+ and μ_j^+ the corresponding functions after the introduction of the policy. The welfare measure I use for this quantitative assessment is the compensating amount of assets that maintain the equivalence between welfares before and after the introduction of the policy reform. In particular, I use three measures of it depending on the level of disaggregation for these measures. First, the aggregate welfare measure is given by Δ solving

$$\begin{aligned} \sum_{j,z} \int_{a,h} \left[u_j(c) V_j^+(a(1 + \Delta), h, \mathbf{z}) \right] \cdot \mu^+(a, h, \mathbf{z}) \cdot \partial a \partial h &= \sum_{j,z} \int_{a,h} V_j(a, h, \mathbf{z}) \cdot \mu(a, h, \mathbf{z}) \cdot \partial a \partial h. \\ \sum_{j,z} \int_{a,h} V_j^+(a(1 + \Delta), h, \mathbf{z}) \cdot \mu^+(a, h, \mathbf{z}) \cdot \partial a \partial h &= \sum_{j,z} \int_{a,h} V_j(a, h, \mathbf{z}) \cdot \mu_j(a, h, \mathbf{z}) \cdot \partial a \partial h. \end{aligned}$$

where

$$\mathcal{W}(\Delta) \equiv \sum_{j,z} \int_{a,h} \left[\sum u_j V_j(a, h, \mathbf{z}) \right] \cdot \mu_j^+(a, h, \mathbf{z}) \cdot \partial a \partial h.$$

$$\mathcal{W}_j(\Delta, \mathbf{x}) \equiv u_j(c_j(\mathbf{x})[1 + \Delta]) + \omega(h_{+,j}(\mathbf{x})) + \beta W_j(a_+, \mathbf{z}).$$

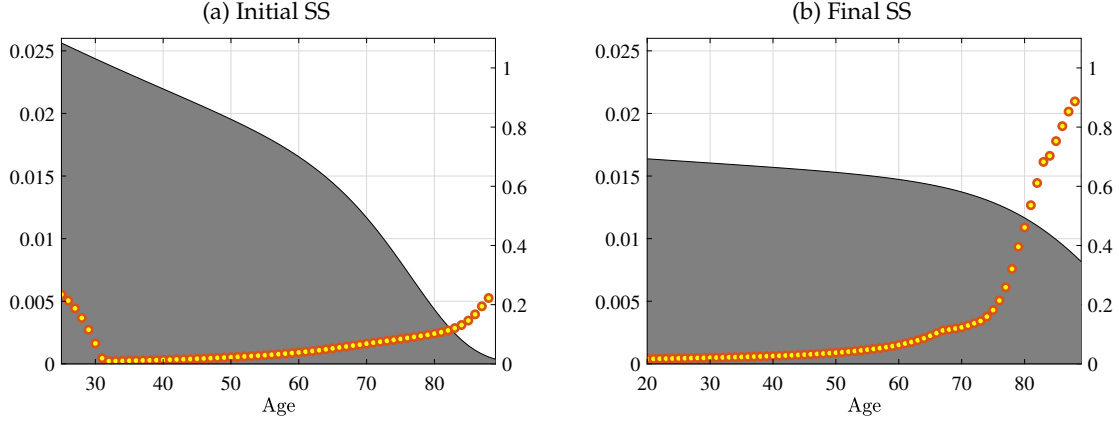
Second, the age-dependent measure of this type of policy reform is Δ_j solving the following equation for each j :

$$\sum_z \int_{a,h} V_j^+(a(1 + \Delta_j), h, \mathbf{z}) \cdot \mu^+(a, h, \mathbf{z}) \cdot \partial a \partial h = \sum_z \int_{a,h} V_j(a, h, \mathbf{z}) \cdot \mu(a, h, \mathbf{z}) \cdot \partial a \partial h.$$

Lastly, to partially capture the distributive

$$\int_h V_j^+(a(1 + \Delta_j(a, \mathbf{z})), h, \mathbf{z}) \cdot \mu^+(a, h, \mathbf{z}) \cdot \partial h = \sum_z \int_{a,h} V_j(a, h, \mathbf{z}) \cdot \mu(a, h, \mathbf{z}) \cdot \partial a \partial h.$$

Figure D.1: Age Distribution and MPC



D.4 Other Calibration Results

E Proofs

To simplify notation, I omit dependences on optimal choices.

E.1 Proof of Lemma B.2

The optimality condition for this policy is $u'(y - \omega) = V'(\omega)$. Taking first and second-order derivatives to this optimality condition yields

$$\omega' = \frac{1}{1+x} \in (0,1) \text{ and } \omega'' = -\frac{x'}{(1+x)^2}$$

where $x \equiv \frac{V''}{u''}$. Thus, $\omega'' > 0$ if $x' < 0$, i.e., V'' rises slower than u'' as income grows. In particular, taking the first-order derivative of x and rearranging yields

$$x' = -\frac{1-\omega'}{\omega} \sigma_V \left[\frac{\rho_V}{\sigma_V} - \frac{\rho_u}{\sigma_u} \right].$$

Using the expressions for ω' and ω'' , the curvature of the asset policy ω can be written as

$$\frac{\omega'' y}{\omega'} = (1-\omega') \sigma_V \frac{\omega'}{\omega/y} \left[\frac{\rho_V}{\sigma_V} - \frac{\rho_u}{\sigma_u} \right].$$

Rearranging yields to (B.4). Moreover, the elasticity of ω can be written as

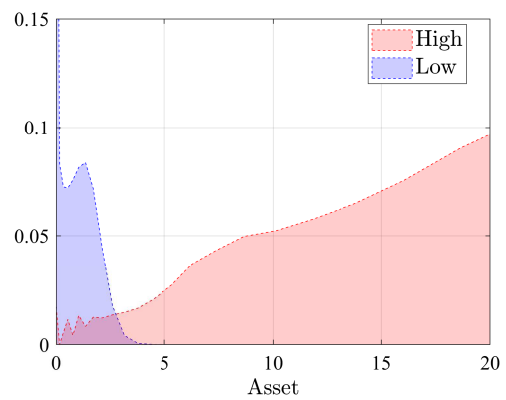
$$\eta_\omega = \frac{y}{\omega + \omega \frac{V''/V'}{u''/u'}} = \frac{y}{\omega + (y - \omega) \cdot \frac{\sigma_V}{\sigma_u}}.$$

Figure D.2: Asset Distribution

(a) Initial SS by working status



(b) Initial SS by income



(c) Final SS by working status



(d) Final SS by income

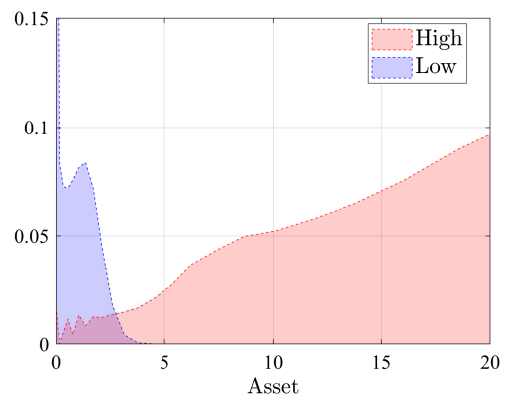
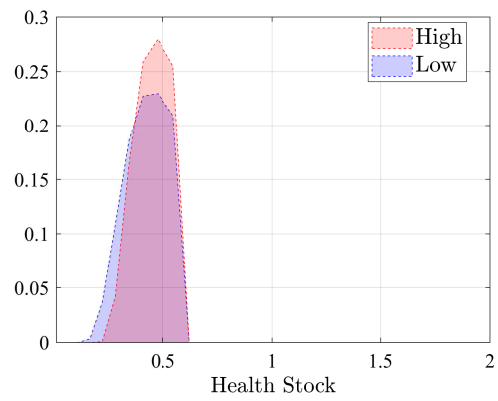


Figure D.3: Health stock Distribution

(a) Initial SS by working status



(b) Initial SS by income



(c) Final SS by working status



(d) Final SS by income

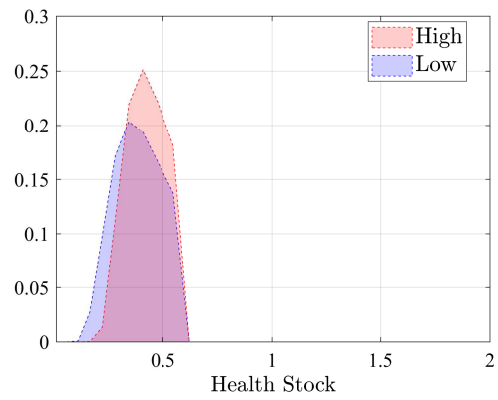
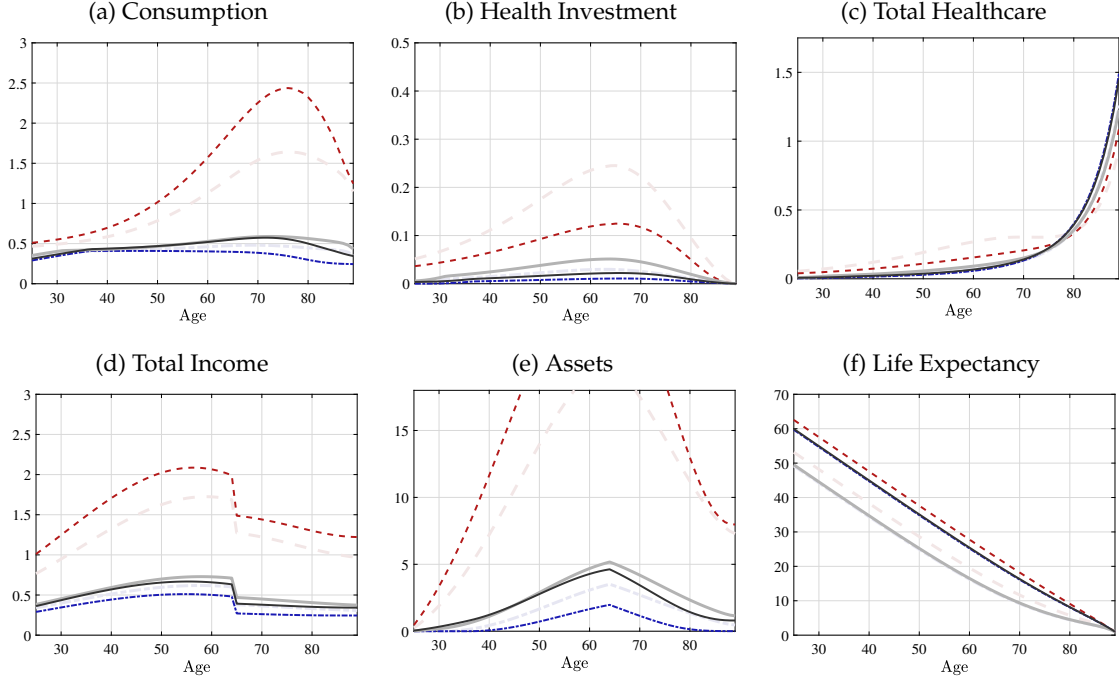


Figure D.4: Individual decisions over the life cycle



E.2 Proof of Lemma B.3

Note that the optimality condition for α is $R \cdot \vartheta_U(R\alpha) = \vartheta_\Psi(\omega - \alpha)$. Taking first- and second-order derivatives to this optimality condition yields

$$\alpha' = \frac{1}{1 + R^2 \cdot \chi} \quad \text{and} \quad \alpha'' = -\frac{R^2 \cdot \chi'}{(1 + R^2 \cdot \chi)^2}$$

where $\chi \equiv \frac{\vartheta'_U}{\vartheta'_\Psi}$ measures the satiation of U relative to that of Ψ . Moreover, from the envelope theorem, the first-order derivative of V is $V' = R\Psi(\lambda)U'(R\alpha)$.

Satiation index for V . Taking consecutive derivatives and omitting dependences yields

$$\begin{aligned} V'' &= R\Psi'U'\lambda' + R^2\Psi U''\alpha' \\ V''' &= R\Psi''U'(\lambda')^2 + R^2\Psi'U''\lambda'\alpha' - R\Psi'U'\alpha'' + R^2\Psi'U''\alpha'\lambda' + R^3\Psi U'''(\alpha')^2 + R^2\Psi U''\alpha'' \end{aligned}$$

Then, using the definitions for concavity and rearranging yields

$$\begin{aligned} \sigma_V &\equiv -\frac{V''\omega}{V'} = -\frac{\Psi'}{\Psi}\lambda'\omega - R\frac{U''}{U'}\alpha'\omega = \frac{\alpha'\omega}{\alpha} \cdot \left(\sigma_U - \vartheta_\Psi \frac{\lambda'}{\alpha'} \alpha \right) \\ &= \frac{\alpha'\omega}{\alpha} \cdot \left(\sigma_U - \vartheta_\Psi \frac{\vartheta'_U}{\vartheta'_\Psi} R^2 \alpha \right) = \frac{\alpha'\omega}{\alpha} \cdot \left(\sigma_U + \frac{\vartheta_\Psi h}{\vartheta'_\Psi h / \vartheta_\Psi} |\eta_{\vartheta_U}| \right) \\ &= \eta_\alpha \cdot \sigma_U \cdot \left(1 - \phi \cdot \frac{|\eta_{\vartheta_U}|}{\sigma_U} \right) \end{aligned}$$

where $\phi \equiv \frac{\eta_\Psi}{\sigma_\Psi + \eta_\Psi}$. Proceeding similarly,

$$\begin{aligned}\frac{V'''}{V'} &= \frac{\Psi''}{\Psi} (\lambda')^2 + 2R \frac{\Psi' U''}{\Psi U'} \lambda' \alpha' + R^2 \frac{U'''}{U'} (\alpha')^2 + \left(R \frac{U''}{U'} - \frac{\Psi'}{\Psi} \right) \alpha'' \\ \frac{V'''}{V'} &= -\sigma_\Psi \eta_\Psi \left(\frac{\lambda'}{\lambda} \right)^2 - 2\eta_\Psi \sigma_U \frac{\lambda' \alpha'}{\lambda \alpha} + \rho_U \sigma_U \left(\frac{\alpha'}{\alpha} \right)^2 - (\sigma_U + \eta_U) \frac{\alpha''}{\alpha}\end{aligned}$$

This expression can be rewritten as

$$\begin{aligned}\rho_V \sigma_V &= \rho_U \sigma_U \eta_\alpha^2 + |\eta_{\theta_U}| \sigma_\alpha \eta_\alpha - \sigma_U \eta_\Psi \eta_\alpha^2 \left(\frac{\sigma_\Psi}{\sigma_U} \left(\frac{\eta_\lambda}{\eta_\alpha} \right)^2 + 2 \frac{\eta_\lambda}{\eta_\alpha} \right) \\ &= \eta_\alpha^2 \rho_U \sigma_U \left(1 + \frac{|\eta_{\theta_U}| \sigma_\alpha}{\rho_U \sigma_U \eta_\alpha} - \frac{\eta_\Psi \eta_\lambda}{\rho_U \eta_\alpha} \left(\frac{\sigma_\Psi \eta_\lambda}{\sigma_U \eta_\alpha} + 2 \right) \right) \\ &= \eta_\alpha^2 \rho_U \sigma_U \left(1 + \frac{1}{\rho_U} \left[\frac{|\eta_{\theta_U}| \sigma_\alpha}{\sigma_U \eta_\alpha} - \eta_\Psi \frac{\eta_\lambda}{\eta_\alpha} \left(\frac{\sigma_\Psi \eta_\lambda}{\sigma_U \eta_\alpha} + 2 \right) \right] \right)\end{aligned}$$

As a result, the absolute satiation index is given by

$$S_V = S_U \frac{\left(1 - \phi \frac{|\eta_{\theta_U}|}{\sigma_U} \right)^2}{1 + \frac{1}{\rho_U} \left[\frac{|\eta_{\theta_U}| \sigma_\alpha}{\sigma_U \eta_\alpha} - \eta_\Psi \frac{\eta_\lambda}{\eta_\alpha} \left(\frac{\sigma_\Psi \eta_\lambda}{\sigma_U \eta_\alpha} + 2 \right) \right]}.$$

Curvature of α . Taking first-order derivative to χ returns

$$\chi' = \frac{\partial''_U}{\partial'_\Psi} R \alpha' - \lambda' \frac{\partial'_U}{\partial'_\Psi} \frac{\partial''_\Psi}{\partial'_\Psi} = \frac{\lambda'}{R} \left(\frac{\partial''_U}{\partial'_\Psi} \frac{1}{\chi} - R \frac{\partial'_U}{\partial'_\Psi} \frac{\partial''_\Psi}{\partial'_\Psi} \right) = \frac{\lambda'}{R} \frac{\partial'_U}{\partial_U} \left(1/S_U^{\text{rel}} - 1/S_\Psi^{\text{rel}} \right)$$

Then, rearranging yields

$$\sigma_\alpha = -\frac{\alpha'' \omega}{\alpha'} = R^2 \chi' \cdot \alpha' \omega = \lambda' \cdot \eta_\alpha \cdot |\eta_{\theta_U}| \cdot \left(1/S_U^{\text{rel}} - 1/S_\Psi^{\text{rel}} \right).$$

Moreover, the elasticity of α can be written as

$$\eta_\alpha = \frac{\omega}{\alpha + R \alpha \frac{\partial'_U / \partial_U}{\partial'_\Psi / \partial_\Psi}} = \frac{\omega}{\alpha + (\omega - \alpha) \cdot \frac{|\eta_{\theta_U}|}{|\eta_{\theta_\Psi}|}}.$$

E.3 Proof of Proposition B.1

Given the parametric assumptions, $\phi(h) = \frac{\zeta H(h)[1-\Psi(h)]}{\sigma_H + \zeta H(h)\Psi(h)}$ and $\eta_\Psi(h) = \zeta H(h)[1-\Psi(h)]$. Then, as the total resources $\omega \rightarrow 0$, health investments $h \rightarrow 0$ and $\lim_{h \rightarrow 0} \phi(h) = \lim_{h \rightarrow 0} \eta_\Psi(h) = 0$. As we know that $\lim_{c \rightarrow c_{\min}} |\eta_{\theta_U}(c)| = \infty$, then $S_V \simeq 0 < S_U = \frac{\Sigma}{\Sigma+1}$. Since $|\eta_{\theta_U}|$ shrinks and η_Ψ rises as ω grows, then eventually S_V surpasses S_U .

E.4 Proof of Proposition B.3

Recall that the concavity of α is pinned down by $\sigma_\alpha = \lambda' \cdot \eta_\alpha \cdot \kappa_U \cdot \left(1/S_U^{\text{rel}} - 1/S_\Psi^{\text{rel}}\right)$. As total resources ω shrinks, $c \rightarrow 0$ and $h \rightarrow 0$. So, we know that $\sigma_\alpha > 0$ for small levels of ω . Instead, for high levels of ω , $c \rightarrow \infty$ and $h \rightarrow \infty$, so that $\sigma_\alpha < 0$. Due to continuity of σ_α , there exists a threshold resource level $\bar{\omega}$ such that $\sigma_\alpha(\omega) > 0$ for all $\omega < \bar{\omega}$ and $\sigma_\alpha(\omega) < 0$, otherwise.

E.5 Proof of Proposition C.1

Full-Redistribution. The Lagrangian for the problem is

$$\begin{aligned} & \sum_{t=0}^{\infty} \tilde{\Phi}_t \int_{i \in \mathcal{N}_t} \phi_t^i \left[u(c_{y,t}^i) + \Psi_{t+1}(h_t^i) U(c_{o,t+1}^i) \right] \partial i + \tilde{\Phi}_{-1} \int_{i \in \mathcal{N}_{-1}} \phi_{-1}^i \Psi_0(h_{-1}^i) U(c_{o,0}^i) \partial i \\ & + \sum_{t=0}^{\infty} \lambda_t \left[F_t(K_t, N_{y,t}) + (1 - \delta)K_t - \int_{i \in \mathcal{N}_t} (c_{y,t}^i + h_t^i) \partial i - \int_{i \in \mathcal{N}_{t-1}} \Psi_t(h_{t-1}^i) [c_{o,t}^i + M(h_{t-1}^i)] \partial i - K_{t+1} \right] \end{aligned}$$

where $\tilde{\Phi}_t \equiv \Phi_t / N_{y,t}$. Rearranging the optimality conditions with respect to h_t^i , $c_{y,t}^i$, and $c_{o,t+1}^i$ for $i \in \mathcal{N}_t$ yields $\tilde{\Phi}_t \phi_t^i u'(c_{y,t}^i) = \lambda_t$, $\beta \tilde{\Phi}_t \phi_t^i U'(c_{o,t+1}^i) = \lambda_{t+1}$, and

$$\Psi'_{t+1}(h_t^i) c_{o,t+1}^i [\Theta(c_{o,t+1}^i) - 1] - [\Psi'_{t+1}(h_t^i) M(h_t^i) + \Psi_{t+1}(h_t^i) M'(h_t^i)] = \frac{\lambda_t}{\lambda_{t+1}}.$$

Due to the concavity of Ψ and convexity of $\Psi \cdot M$, the left-hand side is decreasing in h , leading to a unique solution for a given $c_{o,t+1}^i$. Thus, if Pareto weights are uniform, the solution is the same for all individuals.

Optimality Conditions. The planning problem is: Given the initial stock of capital k_0 and a path for fertility and healthcare technology $\{\Psi_t\}_{t \geq 0}$, the planner chooses $\{h_t, c_{y,t}, c_{o,t}, k_{t+1}\}_{t \geq 0}$ to maximize

$$\sum_{t=0}^{\infty} \Phi_t \left[u(c_{y,t}) + \Psi_{t+1}(h_t) U(c_{o,t+1}) \right] + \Phi_{-1} \Psi_0(h_{-1}) U(c_{o,0}) \quad (\text{E.1})$$

subject to the resource constraint

$$f_t(k_t) + (1 - \delta)k_t = c_{y,t} + h_t + \frac{\Psi_t(h_{t-1})}{1 + n_t} [c_{o,t} + M(h_{t-1})] + (1 + n_{t+1})k_{t+1}. \quad (\text{E.2})$$

where $f_t(k) \equiv F_t(k, 1)$ and $k_t \equiv K_t / N_{y,t}$. The Lagrangian is

$$\begin{aligned} & \sum_{t=0}^{\infty} \Phi_t \left[u(c_{y,t}) + \Psi_{t+1}(h_t) U(c_{o,t+1}) \right] + \Phi_{-1} \Psi_0(h_{-1}) U(c_{o,0}) \\ & + \sum_{t=0}^{\infty} \lambda_t \left[f_t(k_t) + (1 - \delta)k_t - c_{y,t} - h_t - \frac{\Psi_t(h_{t-1})}{1 + n_t} [c_{o,t} + M(h_{t-1})] - (1 + n_{t+1})k_{t+1} \right]. \end{aligned}$$

The first-order conditions are (i) with respect to young's consumption, c_t^y : $\Phi_t u_t' = \lambda_t$, (ii) with respect to old's consumption, c_t^o : $\Phi_{t-1} U_t' = \frac{\lambda_t}{1+n_t}$, (iii) with respect to health stock, h_t :

$$\Phi_t \Psi'_{t+1} U_{t+1} = \lambda_t + (\Psi'_{t+1}(c_{o,t+1} + M_{t+1}) + \Psi_{t+1} M'_{t+1}) \frac{\lambda_{t+1}}{1+n_{t+1}},$$

and (iv) with respect to physical capital, k_{t+1} : $\lambda_t = R_{t+1} \frac{\lambda_{t+1}}{1+n_{t+1}}$ where $R_{t+1} \equiv 1 - \delta + f'_{t+1}$. Combining (i) and (ii) yields $\phi_t^{-1}(1+n_t) = \frac{u_t'}{U_t'}$. Moreover, from (i), (ii) and (iii), we have

$$\left(1 + \frac{\Psi'_{t+1}}{R_{t+1}}(c_{o,t+1} + M_{t+1})\right) \frac{u_t'}{\Psi_{t+1} U_{t+1}'} = \frac{\Psi'_{t+1} U_{t+1}}{\Psi_{t+1} U_{t+1}'} - M'_{t+1}.$$

Combining (i), (ii) and (iv) yields the standar Euler equation: $u_t' = \frac{1}{\Psi_{t+1}} R_{t+1} \Psi_{t+1} U_{t+1}'$.

E.6 Proof of Proposition 6

Given $\phi \in (0, 1)$, the stationary planner's allocation is given by $\{c_y, c_o, h, k\}$ solving the following system of four non-linear equations

$$\begin{aligned} \frac{u'(c_y)}{U'(c_o)} &= \phi^{-1}(1+n) = 1 - \delta + f'(k) = \Psi'(h)c_o \left(\frac{U(c_o)}{U'(c_o)c_o} - 1 \right) - G'(h) \\ f(k) - (n + \delta)k &= c_y + h + \frac{\Psi(h)c_o + G(h)}{1+n}. \end{aligned}$$

Constant Stock of Physical Capital. A first implication of these set of optimal conditions is that the stock of capital is fully pinned down by capital productivity f' , depreciation rate r , population growth rate n , and the planner's intergenerational preferences ϕ . More importantly, the aggregate stock of capital and the real interest rate do not depend on the survival technology in the economy.

Consumption and Health Investment Allocation. Under the assumptions over Ψ and M , these conditions can be rewritten as

$$\text{cte}_0 = \frac{U'(c_o)}{u'(c_y)}, \quad \text{cte}_1 = \Psi'(h)c_o \left(\frac{U(c_o)}{U'(c_o)c_o} - 1 \right) - G'(h), \quad \text{cte}_2 = c_y + h + \frac{\Psi(h)c_o + G(h)}{1+n}$$

where cte_0 , cte_1 , and cte_2 can be treated as irrelevant constant for the following analysis. Taking derivative assuming that the only demographic shift comes from survival rates (captured by parameter ψ governing Ψ) yields

$$\hat{c}_y = \frac{\Sigma}{\sigma} \cdot \hat{c}_o \tag{E.3}$$

$$0 = \frac{\Theta}{\Theta - 1} \Sigma \cdot \hat{c}_o - \omega_h \cdot \hat{h} + \omega_\psi \cdot \partial \psi \tag{E.4}$$

$$0 = c_y \cdot \hat{c}_y + \text{abs}_c \cdot \hat{c}_o + \left(1 + \phi^{-1} \omega_c\right) h \cdot \hat{h} + \text{abs}_\psi \cdot \partial \psi \tag{E.5}$$

where $\hat{x} \equiv \frac{\partial x}{\partial \psi}$, $\Sigma(c) \equiv -\frac{U''(c)}{U'(c)}c$ and $\sigma(c) \equiv -\frac{u''(c)}{u'(c)}c$ are the relative risk aversion coefficients, and $\omega_c > 0$, $\omega_h > 0$, $\text{abs}_c > 0$, $\text{abs}_\psi > 0$, and $\omega_\psi \geq 0$ are all parameters defined in Appendix C.2. Rearranging the previous system and solving for \hat{h} yields $\hat{h} = \Omega(\psi) \cdot \partial\psi$ where

$$\Omega(\psi) = \frac{\frac{\Theta-1}{\Theta} \frac{\omega_\psi}{\Sigma} \left(\frac{\Sigma}{\sigma} c^y + \text{abs}_c \right) - \text{abs}_\psi}{\frac{\Theta-1}{\Theta} \frac{\omega_h}{\Sigma} \left(\frac{\Sigma}{\sigma} c^y + \text{abs}_c \right) + (1 + \phi^{-1} \omega_c) h}$$

The function $\Omega(\psi)$ may take positive or negative values depend on the level of ψ and the nature of it. Then, the response of the allocation depends on the nature of the shift in the survival rate.

Increment in Survival Level and Productivity in Health Investment. Suppose the survival rate experiences a shift such that $\frac{\partial \Psi}{\partial \psi}, \frac{\partial \Psi'}{\partial \psi} > 0$. Under this shift, $\omega_\psi > 0$ and $\hat{c}_o \geq 0$ is not possible as it would require $\hat{h} > 0$ from equation Equation (E.4), but $\hat{h} < 0$ from equation Equation (E.5). Hence, consumption declines for all agents, i.e., $\hat{c}_o, \hat{c}_y < 0$.

Moreover, $\Omega(\psi) > 0$ if and only if $\frac{\Theta-1}{\Theta} \frac{\omega_\psi}{\Sigma} \left(\frac{\Sigma}{\sigma} c^y + \text{abs}_c \right) > \text{abs}_\psi$. The term $\frac{\Theta-1}{\Theta} \frac{\omega_\psi}{\Sigma} \left(\frac{\Sigma}{\sigma} c^y + \text{abs}_c \right)$ captures the increment in health investment's productivity, while the term abs_ψ captures the increment in the elderly population leading to negative wealth effects in the whole economy. In general, there is a race between these two forces under this type of survival/medical technology progress. One might expect that a "well-behaved" survival technological progress features a decreasing ω_ψ . Therefore, in general, there exists a parameter level $\bar{\psi}$ such that $\Omega(\psi) > 0$ for all $\psi < \bar{\psi}$ and $\Omega(\psi) < 0$ otherwise.

Two examples can be provided for this type of technology improvement. First, consider a neutral shift in Ψ , i.e., a movement in a scale parameter ψ in $\Psi(h) = \psi \Psi_s(h)$. This shift has the property of affecting survival rates uniformly, i.e., $\frac{\partial \Psi}{\partial \psi} \frac{\psi}{\Psi} = 1$ for all h . Moreover, in this case $\omega_\psi = 1/\psi$, satisfying the desire property. Second, suppose Ψ is given by (7). A shift in $\bar{\psi}_h$ leads to an increment in survival and increment in the productivity in health investment. Under these shifts, ω_ψ is larger the lower is $\bar{\psi}_h$.

Increment in Survival Level but Decline in Productivity in Health Investment. Now, suppose the survival rate experiences a shift such that $\frac{\partial \Psi}{\partial \psi} > 0$ and $\frac{\partial \Psi'}{\partial \psi} < 0$. Under this setting, ω_ψ generally is negative, inducing that $\Omega(\psi) < 0$ for all ψ . An example of this form of technological progress is changes in the parameter $\bar{\psi}$ in (7).

E.7 Proof of Proposition 7

Under the assumptions made in the proposition, given $k_0 = k_-$ and $h_{-1} = h_-$, the optimal planner's allocation solves the dynamic system given by equations (C.5)-(C.8) plus the appropriate transversality condition. Using results from Appendix C.2, the dynamic allocation can be approximated as percentage deviations from the steady state, \hat{x} for any variable x . Since the change in permanent in our experiment, it is simpler to get an analytical solution by assuming the system is perturbed around the final/new steady state, i.e., $\hat{x}_t \equiv \frac{x_t - x_\infty}{x_\infty}$. Under this assumption, for any (exogenous) variable changing permanently, $\hat{x}_{-1} \neq 0$ and $\hat{x}_t = 0$ for $t \geq 0$.

The planner's allocation deviations are determined by

$$\widehat{c}_{o,t} = \begin{cases} \left(\Sigma^{-1} \omega_k + \frac{(1-\phi\theta)(1+n-\varsigma)}{\phi\varrho} \right) \cdot \widehat{k}_1 & t = 0 \\ \frac{(1-\phi\theta)(1+n-\varsigma)}{\phi\varrho} \cdot \widehat{k}_t & t \geq 1 \end{cases}$$

$$\widehat{k}_t = \underline{\theta}^{t-1} \cdot \widehat{k}_1 \text{ for all } t \geq 1$$

$$\widehat{h}_t = \frac{\Sigma}{\omega_h} \cdot \widehat{c}_{o,t} + \frac{\omega_c \Sigma}{\omega_h} \cdot \widehat{c}_{o,t+1}$$

with the initial capital response given by

$$\widehat{k}_1 = - \frac{\phi^{-1} \omega_c \frac{h}{k} \cdot \widehat{h}_{-1}}{\Sigma^{-1} \varrho \omega_k + (1+n-\varsigma) \left(1 + \phi^{-1} (1-\phi\theta) \frac{\varrho}{q} \right)}$$

where $\omega_k, \varrho, q > 0$ and $\underline{\theta} \in (0, 1)$ are parameters all defined in Appendix C.2. Note that all the dynamism in the system comes from the permanent response in health investment, $\widehat{h}_{-1} \equiv \frac{h_{\text{old}} - h_{\text{new}}}{h_{\text{new}}}$, which is characterized in Appendix E.6.

Increment in Survival Level and Productivity in Health Investment. The economically relevant case for this type of technological improvement is one in which $h_{\text{old}} < h_{\text{new}}$, that is, $\widehat{h}_{-1} < 0$, which generates $\widehat{k}_t > 0$, $\widehat{c}_{o,t} > 0$ and $\widehat{h}_t > 0$. Under this scenario, the planner's allocation converges toward the new steady state from above. Because the steady-state level of the capital stock does not change, this implies that $k_1 > k$ and that capital then converges monotonically back to the same steady state.

Since the steady-state stock of health is higher, $\widehat{h}_t > 0$ implies that health overshoots on impact: $h_0 > h_{\text{new}} > h_{\text{old}}$, after which it converges smoothly back to the higher new steady-state level. Finally, because both the stock of capital and the stock of health increase at $t = 0$, old-age consumption initially falls, so that $c_{o,0} < c_{o,\text{old}}$. However, $\widehat{c}_{o,t} > 0$ implies that $c_{o,\text{old}} > c_{o,0} > c_{o,\text{new}}$, and consumption then converges monotonically toward the new, lower steady-state level.

Increment in Survival Level but Decline in Productivity in Health Investment. In this case, $\widehat{h}_{-1} > 0$, generating $\widehat{k}_t < 0$, $\widehat{c}_{o,t} < 0$ and $\widehat{h}_t < 0$. This means that planner's allocation converges toward the new steady state from below. This means that $k_1 < k$. Moreover, since the stock of health is smaller, $\widehat{h}_t < 0$ means that the stock of health overshoots at $t = 0$, i.e., $h_{\text{old}} > h_{\text{new}} > h_0$, and then reverts back to the lower new steady state level. As the stock of capital and health decline at $t = 0$, consumption for olds can fall or rise and the final outcome depends as usual in the how powerful is the elasticity of intertemporal substitution.

E.8 Proof of Proposition 8

Under the perfect redistribution system, the cash-on-hand for all young agents of generation t is the same, $z^i w_t + b_{q,t}^i - \tau_t^i = \bar{z}_t w_t + \bar{b}_{q,t}$, so that consumption and investment decisions are the same for them. Given the complete medical insurance, no agent faces idiosyncratic risk for medical expenses net of subsidies which, under the government intervention, is given by $M(h^i) - s_{t+1}^i =$

$M(h^i) - \bar{s}_t$. Moreover, the annuities offered by the government dominate the agent's saving decisions as $R_{t+1}/\Psi_{t+1} \geq R_{t+1}$. Then, the first-order condition for individual's demand of physical capital is given by (21) and for health capital is

$$(1 + \tau_{h,t})u'_t = U'_{t+1} \left[\Psi'_{t+1} \frac{U_{t+1}}{U'_{t+1}} - \Psi_{t+1} M' \right].$$

Combining this equation with condition (22) yields (23). The government budget constraint is

$$B_{g,t+1} + N_{y,t}(\bar{\tau}_t + \tau_{h,t}\bar{h}_t) = N_{o,t}s_t + R_t B_{g,t}.$$

Making $B_{g,t} = 0$ for all $t \geq 0$, the budget constraint of the government imposes the following restrictions on average transfers: $\bar{\tau}_t + \tau_{h,t}^h \bar{h}_t^* = d_t \bar{s}_t$ where $\bar{\tau}_t$ (or \bar{s}_t) achieves the appropriate intergenerational transfer.

E.9 Proof of Proposition C.2

Let λ_t be the Lagrange multiplier for the resource constraint (20) and ν_t the multiplier for the IC constraint (24). Then, the Lagrangian for this problem can be written as

$$\begin{aligned} & \sum_{t=0}^{\infty} \Phi_t \{ u_t - \nu_t u'_t + \Psi_{t+1} (U_{t+1} + \nu_t R_{t+1} U'_{t+1}) \} + \Phi_{-1} \Psi_0 (h_{-1}) U_0 \\ & + \sum_{t=0}^{\infty} \lambda_t \left\{ f_t(k_t) + (1 - \delta)k_t - (1 + n_{t+1})k_{t+1} - c_{y,t} - h_t - \frac{\Psi_t(h_{t-1})}{1 + n_t} [c_{o,t} + M(h_{t-1})] \right\}. \end{aligned}$$

The first-order condition are: (i) with respect to young's consumption, $c_{y,t}$:

$$\Phi_t (u'_t - \nu_t u''_t) = \lambda_t,$$

(ii) with respect to old's consumption, $c_{o,t}$:

$$\Phi_{t-1} (U'_t + \nu_{t-1} R_t U''_t) = \frac{\lambda_t}{1 + n_t} \text{ with } \nu_{-1} = 0,$$

(iii) with respect to health stock, h_t :

$$\Phi_t \Psi'_{t+1} (U_{t+1} + \nu_t R_{t+1} U'_{t+1}) - \Psi_{t+1} (M'_{t+1} + \chi_{t+1}) \frac{\lambda_{t+1}}{1 + n_{t+1}} = \lambda_t,$$

and (iv) with respect to capital stock, k_{t+1} :

$$\nu_t \Phi_t d_{t+1} R'_{t+1} U'_{t+1} + R_{t+1} \frac{\lambda_{t+1}}{1 + n_{t+1}} = \lambda_t.$$

Combining conditions (i) and (ii) yields: For all $t \geq 0$

$$\phi_t^{-1} (1 + n_t) = (1 + \omega_{ig,t}) \cdot \frac{u'_t}{U'_t} \text{ where } \omega_{ig,t} \equiv \frac{-\frac{u''_t}{u'_t} \nu_t - \frac{U''_t}{U'_t} R_t \nu_{t-1}}{1 + \frac{U''_t}{U'_t} R_t \nu_{t-1}}.$$

In turn, combining (iii) and (iv) yields

$$(1 + \omega_{h,t})u'_t = U'_{t+1} \left(\Psi'_{t+1} \frac{U_{t+1}}{U'_{t+1}} - \Psi_{t+1} M'_{t+1} \right)$$

where

$$\omega_{h,t} \equiv \left(1 + \frac{U''_{t+1}}{U'_{t+1}} R_{t+1} v_t \right) \frac{\chi_{t+1}}{R_{t+1}} - v_t \left(\frac{\Psi'_{t+1}}{\Psi_{t+1}} + \frac{u''_t}{u'_t} - M'_{t+1} \frac{U''_{t+1}}{U'_{t+1}} \right).$$

Finally, the FOCs in (iv) can be written as

$$\psi_{t+1} \left(1 - \frac{u''_t}{u'_t} v_t \right) = 1 - v_t \left(-\frac{U''_{t+1}}{U'_{t+1}} R_{t+1} - d_{t+1} \frac{R'_{t+1}}{R_{t+1}} \right).$$

E.10 Proof of Proposition

$$\max_{\{c_{y,t}(\ell), c_{o,t}(\ell), h_t(\ell), k_{t+1}\}_{t \geq 0}} \sum_{t=0}^{\infty} \Phi_t \int [u_t(z) + \Psi_{t+1}(z) U_{t+1}(z)] \partial \mathbb{P}_t + \Phi_{-1} \int \Psi_0(h_{-1}(z)) U(c_{o,0}(z)) \partial \mathbb{P}_{-1}$$

subject to

$$u'(c_{y,t}(z)) = R_{t+1} \Psi_{t+1}(h_t(z)) U'(c_{o,t+1}(z))$$

$$\text{cte} = \frac{\Psi'_{t+1}(h_t(z)) U(c_{o,t+1}(z))}{\Psi_{t+1}(h_t(z)) U'(c_{o,t+1}(z))} - M'(h_t(z))$$

$$f_t(k_t) - (1 - \delta)k_t = \int (c_{y,t}(z) + h_t(z)) \partial \mathbb{P}_t + \frac{1}{1 + n_t} \int \Psi_t(h_{t-1}(z)) (c_{o,t}(z) + M(h_{t-1}(z))) \partial \mathbb{P}_{t-1} + (1 + n_{t+1})k_{t+1}.$$

Let $v_t(z) \mathbb{P}_t(z)$ be the Lagrange multiplier for the Euler equation. Then, the objective function can be rewritten as

$$\begin{aligned} & \sum_{t=0}^{\infty} \Phi_t \int [u_t(z) - v_t(z) u'_t(z) + \Psi_{t+1}(z) \{U_{t+1}(z) + v_t(z) R_{t+1} U'_{t+1}(z)\}] \partial \mathbb{P}_t \\ & + \Phi_{-1} \int \Psi_0(h_{-1}(z)) U(c_{o,0}(z)) \partial \mathbb{P}_{-1} \end{aligned}$$

Now let λ_t be the Lagrange multiplier for the resource constraint and $\theta_{t+1}(z) \Phi_t \mathbb{P}_t(z)$ for the second IC constraint. Then the first order conditions can be written as: (i) with respect to young's consumption, $c_{y,t}(z)$:

$$\Phi_t (u'_t(z) - v_t(z) u''_t(z)) = \lambda_t,$$

(ii) with respect to old's consumption, $c_{o,t}(z)$:

$$\Phi_{t-1} \Psi_t(z) (U'_t(z) + v_{t-1}(z) R_t U''_t(z)) - \Phi_{t-1} \theta_t(z) \frac{\Psi'_t(z)}{\Psi_t(z)} \left(1 - \frac{U_t(z)}{U'_t(z)} \frac{U''_t(z)}{U'_t(z)} \right) = \Psi_t(z) \frac{\lambda_t}{1 + n_t}$$

with $v_{-1}(z) = 0$ and $\theta_{-1}(z) = 0$, (iii) with respect to health stock, $h_t(z)$:

$$\Phi_t \Psi'_{t+1}(z) (U_{t+1}(z) + v_t(z) R_{t+1} U'_{t+1}(z)) - \Psi_{t+1}(z) (M'_{t+1}(z) + \chi_{t+1}(z)) \frac{\lambda_{t+1}}{1 + n_{t+1}}$$

$$-\Phi_t \theta_{t+1}(z) \left(\left(\frac{\Psi''_{t+1}(z)}{\Psi'_{t+1}(z)} - \frac{\Psi'_{t+1}(z)}{\Psi_{t+1}(z)} \right) \frac{\Psi'_{t+1}(z)}{\Psi_{t+1}(z)} \frac{U_{t+1}(z)}{U'_{t+1}(z)} - M''_{t+1}(z) \right) = \lambda_t,$$

and (iv) with respect to capital stock, k_{t+1} :

$$\Phi_t \int v_t(z) \Psi_{t+1}(z) R'_{t+1} U'_{t+1}(z) \partial \mathbb{P}_t + R_{t+1} \frac{\lambda_{t+1}}{1+n_{t+1}} = \lambda_t.$$

Then, this second-best allocation does not feature full redistribution. In particular,

$$U'_t(z) + v_{t-1}(z) R_t U''_t(z) - \tilde{\theta}_t(z) \frac{\Psi'_t(z)}{\Psi_t(z)} \left(1 - \frac{U_t(z)}{U'_t(z)} \frac{U''_t(z)}{U'_t(z)} \right) = \frac{\phi_t}{1+n_t} (u'_t(z) - v_t(z) u''_t(z))$$